

# Moduli stabilization in non-supersymmetric Minkowski vacua with anomalous $U(1)$ symmetry

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**ABSTRACT:** We study how two moduli can be stabilized in a Minkowski/de Sitter vacuum for a wide class of string-inspired Supergravity models with an effective Fayet-like Supersymmetry breaking. It is shown under which conditions this mechanism can be made natural and how it can give rise to an interesting spectrum of soft masses, with a relatively small mass difference between scalar and gaugino masses. In absence of a constant superpotential term, the above mechanism becomes completely natural and gives rise to a dynamical supersymmetry breaking mechanism. Some specific type IIB and heterotic string inspired models are considered in detail.

**KEYWORDS:** Supersymmetry Breaking, Supergravity Models, dS vacua in string theory, Superstring Vacua.

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## 1. Introduction

Stabilizing all moduli in a Minkowski vacuum with low energy supersymmetry (SUSY) breaking is among the most important problems in string theory, being a crucial step to connect string theory with SUSY phenomenology, that is supposed to be the best motivated scenario for new physics beyond the Standard Model. In recent years there has been a tremendous progress on the issue of stabilizing moduli, mostly in Type II string theories and at the supergravity (SUGRA) level. A combination of fluxes for Ramond-Ramond (RR) tensor field strengths and for the Neveu Schwarz-Neveu Schwarz (NSNS) field strength  $H$  has been shown to stabilize the complex structure moduli of the unperturbed compactified space, typically a Calabi-Yau 3-fold for  $\mathcal{N} = 1$  SUSY theories in  $D = 4$  dimensions [1]. Subsequently a “scenario” with the Kähler structure moduli stabilized on a Minkowski/de Sitter (dS) vacuum with SUSY breaking has been introduced by Kachru, Kallosh, Linde and Trivedi (KKLT) [2], assuming a complete decoupling between complex structure and Kähler structure moduli. In addition to this assumption, the authors of [2] invoked an explicit SUSY breaking mechanism (the introduction of  $\bar{D}_3$  branes). Although such explicit breaking does not forbid a quantitative study of some interesting physical quantities, such as soft parameters (see e.g. [3]), it is nevertheless desirable to have a fully satisfactory spontaneous SUSY breaking mechanism, motivated also by the fact that SUSY is a local symmetry at the SUGRA level and hence explicit breakings should be avoided. After the

work of [2], indeed, many works have appeared where the explicit SUSY breaking by  $\bar{D}_3$  branes is replaced by spontaneous  $F$  or  $D$ -term breaking of different kind [4–20].

A well-known, simple and interesting SUSY breaking mechanism is the one by Fayet and based on a Fayet-Iliopoulos (FI) term for a  $U(1)_X$  gauge symmetry [21]. Its simplest implementation requires two charged fields,  $\phi$  and  $\chi$ , with opposite  $U(1)_X$  charges  $q_\phi$  and  $q_\chi$ . The requirement of minimizing the  $D_X$  term in the scalar potential induces one of the charged fields, say  $\phi$ , to get a non-vanishing vacuum expectation value (VEV). If the only relevant superpotential term coupling  $\phi$  and  $\chi$  is linear in  $\chi$ , a simple effective Polonyi-like superpotential term is induced and SUSY is broken because  $F_\chi \neq 0$ . A constant FI term necessarily requires in SUGRA the introduction of non-gauge invariant superpotentials, which do not seem to occur in string theory. On the contrary, field-dependent effective FI terms generally arise due to a non-linear transformation of some modulus  $U$  under a would-be anomalous  $U(1)_X$  gauge symmetry. Moduli stabilization and Fayet-like SUSY breaking mechanisms are hence closely interconnected in string theory and one might wonder if their combined action can efficiently be embedded in a KKLT-set up to provide a spontaneous SUSY breaking mechanism, which also does not need to assume a complete decoupling between moduli stabilization and SUSY breaking, like in [2]. A Fayet-like SUSY breaking mechanism has been already shown to successfully give rise to low energy SUSY breaking on a Minkowski/dS vacuum for a KKLT-like SUGRA model, where the FI modulus is identified with the universal Kähler modulus and  $q_\phi = -q_\chi$  [19]. The resulting soft mass parameters for the visible sector are realistic, but present a moderate hierarchy between gaugino and scalar masses, unless one complicates the model by introducing additional (messenger-like) fields [19]. The main drawback of the model of [19] is the introduction of an unnaturally small mass term  $m\phi\chi$  with  $m \sim O(10^{-11} \div 10^{-12})$ , in addition to the usual KKLT fine-tuning of assuming a tiny constant superpotential term  $w_0$ , which is roughly of the same order as  $m$ . A more satisfactory explanation of the smallness of the  $\chi\phi$  coupling is necessary, mainly because higher-order terms of the form  $c_n(\phi\chi)^n$  with  $n > 1$  will generally lead to a restoration of SUSY.

Aim of this paper is to study in some detail how a Fayet-like SUSY breaking mechanism can be realized in string-derived SUGRA theories. We consider a bottom-up four dimensional  $\mathcal{N} = 1$  SUGRA framework, more general than the Type IIB KKLT-like SUGRA setting, so that our results will be of more general validity. In particular we will study the dynamics of a SUGRA system with two moduli, the FI modulus  $U$  and another neutral modulus  $Z$ , the two charged fields  $\phi$  and  $\chi$ , with arbitrary  $q_\chi$  charge (the  $U(1)_X$  charges are normalized so that  $q_\phi = -1$ ), extra hidden vector-like matter and two/three condensing gauge groups responsible for non-perturbatively generated terms necessary to stabilize  $U$  and  $Z$ . We specify only the schematic form of the Kähler potential, which is taken quite generic. As far as the superpotential is concerned, we consider both the cases of moduli-independent couplings of the form  $Y\chi\phi^{q_\chi}$  and non-perturbatively generated moduli dependent couplings of the form  $Y \exp(-\gamma_U U - \gamma_Z Z)\chi\phi^{q_\chi}$ , where  $\gamma_{U,Z}$  are some unspecified constants. Once the hidden mesons of the condensing gauge groups are integrated out, the superpotential becomes the sum of exponential terms (including a constant term  $w_0$ ) and of the coupling  $\chi\phi^{q_\chi}$ . In order to have as much as possible analytic control on this com-

plicated system, we first look for SUSY vacua when  $\chi$  is decoupled and then we consider its backreaction, which generally gives rise to non-SUSY vacua. In this approximation, the coupling  $\chi\phi^{q_\chi}$  is effectively an “up-lifting” term, required to pass from the AdS SUSY vacuum to a Minkowski/dS one with low-energy SUSY breaking. We find that the last step puts quite severe constraints on the parameter space of the superpotential. In particular, we find that the non-perturbatively generated couplings can alleviate the tuning needed on  $Y$  from 2 to 6 orders of magnitude, depending on the model considered, but taken alone they do not allow us to have  $Y \sim O(1)$ , since the back-reaction induced by the up-lifting term becomes so large that the non-SUSY vacuum either disappears or always remains AdS. Natural values of  $Y$  can however be obtained by assuming that  $|q_\chi| > q_\phi$  by a few, so that  $\langle\phi\rangle^{q_\chi}$  can be responsible for the remaining necessary suppression.

Most of our interest is in the hidden sector dynamics of the theory and hence we will not systematically study how SUSY breaking is mediated to the visible sector or the precise form of the soft parameters, most of which are necessarily model dependent. We just notice that gravity mediation of SUSY breaking is preferred to avoid small moduli masses, linked to the gravitino mass, and that the general pattern of the soft mass terms seem very promising. In particular, the gaugino masses, which in string-inspired models with purely gravity mediated SUSY breaking are often considerably smaller than the non-holomorphic scalar masses, can be naturally made heavier in our set-up, thanks to the presence of two moduli. In presence of the FI modulus only, gauginos can take a mass only by assuming a  $U$ -dependent gauge kinetic function for the visible gauge group. Since  $U$  transforms under a  $U(1)_X$  gauge transformation, anomaly cancellation arguments require that some visible matter field has to be charged under  $U(1)_X$ , which in turn gives rise to heavy non-holomorphic scalar masses, induced by the  $D_X$  term [19]. This problem is now simply solved by assuming that the gauge kinetic function for the visible sector depends on  $Z$  only. Moreover, when  $\gamma_Z \neq 0$ , the back-reaction of the up-lifting term on  $Z$  will give rise to an enhancement of  $F_Z$ , so that eventually gauginos just a few times lighter than the gravitino can be obtained.

The fine-tuning problem to get  $Y \ll 1$ , by means of the flatness condition, is just a reflection of the other fine-tuning problem which requires  $w_0 \ll 1$  to get low-energy supersymmetry breaking. Motivated by the idea of solving this second tuning problem, we have also analyzed the situation in which  $w_0$  vanishes, assuming that some stringy symmetry forbids its appearance. In this case, the moduli stabilization mechanism boils down to a racetrack model [22], where the scale of supersymmetry breaking is dynamically generated. Interestingly enough, the back-reaction of the up-lifting term is milder than before and it is now possible to achieve  $Y \sim O(1)$  with small  $q_\chi$  charges, relying on  $\gamma_{Z,U}$  only. Aside from the usual cancellation of the cosmological constant, this model is hence completely natural. When two moduli are considered, similarly to before, one can have a not too hierarchical spectrum of soft masses, although the gauginos are now a bit lighter than in the models with  $w_0 \neq 0$  considered before.

We have supplemented all the above general considerations by analyzing in some detail three specific models:

- i) an orientifold compactification of type IIB on  $\mathbf{CP}_{[1,1,1,6,9]}^4$ , where  $U$  and  $Z$  are identified with the two Kähler moduli of the compactification manifold ( $w_0 \neq 0$ ),
- ii) an heterotic compactification on a generic Calabi-Yau 3-fold, where  $U$  and  $Z$  are identified with the dilaton and the universal Kähler modulus, respectively ( $w_0 \neq 0$ ),
- iii) the model of i) but now assuming  $w_0 = 0$ .

It should be stressed that, in the spirit of our bottom-up approach, none of these models are actually full-fledged string models, but rather they should be seen as inspiring examples to partially fix the arbitrariness in the choice of the Kähler potential, superpotential and gauge kinetic functions we have at the SUGRA level. In all three cases, we have analytically and numerically studied the main properties of the vacuum, including its (meta)stability. The latter is essentially never a serious issue and all the moduli components are always one or two order of magnitudes heavier than the gravitino. In all three cases we have always estimated under which conditions a classical SUGRA analysis is reliable by considering the effect of the universal  $\alpha'$  correction to the moduli Kähler potential [23, 24].

The paper is organized as follows. In section 2 we introduce our general model and analytically show under which conditions a non-SUSY Minkowski/dS vacuum can be obtained. We also show in subsection 2.2 how an effective description where the  $U(1)_X$  gauge field and the field  $\phi$  are integrated out is very useful to get some understanding about some properties of the theory. In subsection 3 we analyze the models with  $w_0 = 0$ , where SUSY breaking is dynamically generated and the up-lifting term is completely natural. In section 4 a brief discussion on the soft mass terms is reported. Section 5 is devoted to the analysis of the example of models i), ii) and iii) mentioned earlier. Section 6 contains our conclusions. Throughout all the paper we use units in which the reduced Planck mass  $M_P = 1$ .

## 2. General two-moduli model

The SUGRA model we consider consists of two moduli multiplets  $U$  and  $Z$ , a hidden gauge group of the form  $G_1 \times G_2 \times U(1)_X$ , with  $G_1$  and  $G_2$  non-abelian factors, massless matter charged under  $U(1)_X$  and under  $G_1$  or  $G_2$  (but not both) and finally two chiral multiplets  $\phi$  and  $\chi$ , charged under  $U(1)_X$  and singlets under  $G_1 \times G_2$ . For concreteness, we take  $G_i = SU(N_i)$  ( $i = 1, 2$ ) and consider  $N_{f_i}$  quarks  $Q_i$  and  $\tilde{Q}_i$  in the fundamental and anti-fundamental representations of  $G_i$ . This is the field content of our model. We assume that the  $U(1)_X$  gauge symmetry is “pseudo”-anomalous, namely that such symmetry is non-linearly realized in one of the two moduli multiplets,  $U$ , the latter mediating a generalized Green-Schwarz mechanism [25]. We normalize the  $U(1)_X$  charges so that  $q_\phi = -1$  and take  $q_{Q_i} + q_{\tilde{Q}_i} > 0$ ,  $q_\chi > 0$ , with the same  $U(1)_X$  charge for all flavours, for simplicity. The model is finally specified by the Kähler potential, superpotential and gauge kinetic functions. Omitting for simplicity flavour and color indices, the full Kähler potential  $K_{\text{Tot}}$  and superpotential  $W_{\text{Tot}}$  are a sum of a visible and hidden sector,  $W_{\text{Tot}} = W_v + W_h$  and  $K_{\text{Tot}} = K_v + K_h$ , where

$$K_v = \alpha_{iv} Q_{iv}^\dagger e^{2q_{iv} V_X + V_v} Q_{iv} \tag{2.1}$$

represents the Kähler potential of the visible sector, with  $i$  running over all visible fields, and we have schematically denoted by  $Q_{iv}$  and  $V_v$  all the visible chiral fields and vector superfields. For simplicity, we have taken  $K_v$  to be diagonal in the visible sector fields. We do not specify the visible superpotential  $W_v$  because it will never enter in our considerations. The hidden sector Kähler and superpotential terms read<sup>1</sup>

$$K_h = K_M + \alpha_\phi \phi^\dagger e^{-2V_X} \phi + \alpha_\chi \chi^\dagger e^{2q_\chi V_X} \chi + \sum_{i=1,2} \alpha_i \left( Q_i^\dagger e^{V_i + 2q_{Q_i} V_X} Q_i + \tilde{Q}_i^\dagger e^{-V_i + 2q_{\tilde{Q}_i} V_X} \tilde{Q}_i \right), \quad (2.2)$$

$$W_h = w_0 + Y(U, Z, \phi) \phi^{q_\chi} \chi + \sum_{i=1,2} c_i(U, Z, \phi) Q_i \tilde{Q}_i \phi^{q_{Q_i} + q_{\tilde{Q}_i}} + \sum_{i=1,2} \eta_i (N_i - N_{if}) \left( \frac{\Lambda_i(U, Z)^{3N_i - N_{if}}}{\det(Q_i \tilde{Q}_i)} \right)^{\frac{1}{N_i - N_{if}}}. \quad (2.3)$$

The holomorphic gauge kinetic functions in the hidden sector are taken to be

$$f_i(U, Z) = n_i U + m_i Z + p_i, \quad f_X(U) = n_X U. \quad (2.4)$$

Several comments are in order. The Kähler potentials (2.1) and (2.2) are supposed to be the first terms in an expansion in the matter fields up to quadratic order:  $K_M, \alpha_{iv}, \alpha_i, \alpha_\phi$  and  $\alpha_\chi^2$  are generally real functions of  $U + U^\dagger - \delta V_X$  and  $Z + Z^\dagger$ . In  $K_h, V_i$  and  $V_X$  denote the vector superfields associated to the non-abelian groups  $G_i$  and  $U(1)_X$ , respectively,  $K_M$  is the Kähler potential for the  $U$  and  $Z$  moduli and  $\delta$  is the Green-Schwarz coefficient. The form of the latter is uniquely fixed by gauge invariance to be

$$\delta = \frac{(q_{Q_1} + q_{\tilde{Q}_1}) N_{1f}}{4\pi^2 n_1} = \frac{(q_{Q_2} + q_{\tilde{Q}_2}) N_{2f}}{4\pi^2 n_2}. \quad (2.5)$$

In the superpotential (2.3), we have allowed for an arbitrary constant term  $w_0$  that is supposed to be the left-over of all the remaining fields which are typically present in any explicit string model — such as the complex structure moduli in KKLT-like compactifications — and assumed of having been integrated out.<sup>3</sup> We assume that  $w_0$  is tiny in Planck units, in order to give rise to a light enough gravitino mass.<sup>4</sup> We will later discuss the case in which  $w_0$  exactly vanishes. The hidden Yukawa couplings  $Y$  and  $c_i$  in  $W_h$  are assumed to generally depend on both moduli. Due to the Peccei-Quinn symmetries associated to  $\text{Im } U$  and  $\text{Im } Z$ , the only allowed moduli dependence is exponential. The superpotential (2.3) is manifestly  $G_i$  invariant, whereas the  $U(1)_X$  invariant is less transparent. Under a  $U(1)_X$  super-gauge transformation with parameter  $\Lambda$ , one has  $\delta_X V_X = -i(\Lambda - \bar{\Lambda})/2$ ,  $\delta_X U = i\delta\Lambda/2$

<sup>1</sup>See also [10] where a similar analysis with a single modulus in a KKLT context has been done.

<sup>2</sup>Notice that for simplicity we have taken the same moduli dependent functions  $\alpha_i$  for the hidden quarks and anti-quarks.

<sup>3</sup>Strictly speaking, these fields can also generate a constant Kähler potential term. For simplicity we neglect it, since it just corresponds to a rescaling of the gravitino mass.

<sup>4</sup>The gravitino mass might also be suppressed by a large negative Kähler potential term, like in the large-volume models of [26], so that  $w_0 \sim O(1)$  can be taken. We do not consider this possibility here.

and  $\delta_X \Phi = iq_\Phi \Lambda \Phi$  for any charged multiplet  $\Phi$ . Gauge invariance constraints then the couplings  $Y$  and  $Z$  to depend on  $U$  by means of the gauge invariant combination  $\exp(-U)\phi^{\delta/2}$ . We then parameterize

$$Y(U, Z, \phi) = Y \phi^{\gamma_U \delta/2} e^{-\gamma_U U - \gamma_Z Z}, \quad c_i(U, Z, \phi) = c_i \phi^{\eta_{i,U} \delta/2} e^{-\eta_{i,U} U - \eta_{i,Z} Z}, \quad (2.6)$$

with  $Y$  a constant and  $c_i$  constant matrices (in flavour space). The phenomenological coefficients  $\gamma_{U/Z}$  and  $\eta_{i,U/Z}$  are non-vanishing for non-perturbatively generated Yukawa couplings only. The last term in  $W_h$  is the non-perturbatively generated superpotential term appearing in  $\mathcal{N} = 1$  theories for  $N_f < N$  [27]. We have found convenient to introduce the factors  $\eta_{1,2} = \pm 1$  in eq. (2.3), which will allow us to set to zero the imaginary parts of  $U$  and  $Z$ . The dynamically generated scales  $\Lambda_i(U, Z)$  are field-dependent and follows from the holomorphic gauge kinetic functions (2.4). From eq. (2.4) we have

$$g_i^{-2} = \text{Re } f_i(U, Z), \quad g_X^{-2} = \text{Re } f_X(U), \quad (2.7)$$

and

$$|\Lambda_i(U, Z)| = e^{-\frac{8\pi^2}{g_i^2(3N_i - N_{if})}}. \quad (2.8)$$

The coefficients  $n_X$ ,  $n_i$ ,  $m_i$  and  $p_i$  in eq. (2.4) are model dependent constants, which we keep generic for the moment. Just for simplicity of the analysis, we have assumed that the  $U(1)_X$  factor depends only on the  $U$  modulus. It is straightforward to check that the non-perturbative superpotential terms in eq. (2.3) are  $U(1)_X$  gauge-invariant provided the two equalities in eq. (2.5) are satisfied.

We will mostly be interested in the dynamics of the hidden sector of the theory, assuming that all visible fields vanish. The scalar potential of the theory has the usual SUGRA form given by  $V = V_F + V_D$ , with

$$V_F = e^{K_h} \left( K_h^{I\bar{J}} D_I W_h \overline{D_{\bar{J}} W_h} - 3|W_h|^2 \right), \quad (2.9)$$

$$V_D = \sum_{i=1,2} \frac{1}{2\text{Re } f_i} D_i^2 + \frac{1}{2\text{Re } f_X} D_X^2. \quad (2.10)$$

In eq. (2.9),  $I, J$  run over the hidden chiral multiplets  $Q_i, \tilde{Q}_i, \phi, \chi, U, Z$ ,  $D_I W_h = \partial_I W_h + (\partial_I K_h) W_h \equiv F_I$  is the Kähler covariant derivative and  $K_h^{I\bar{J}}$  is the inverse Kähler metric. In eq. (2.10),  $D_i$  and  $D_X$  are the D-terms associated to the  $G_i$  and  $U(1)_X$  isometries of  $K$ , generated by Killing vectors  $X_i$  and  $X_X$ , and are given by (omitting gauge indices)

$$D_i = \frac{X_i^I F_I}{W} = X_i^I \partial_I K, \quad D_X = \frac{X_X^I F_I}{W} = X_X^I \partial_I K, \quad (2.11)$$

where the second equalities in the two expressions above apply for a gauge invariant super-

potential. The explicit form of the D-terms is<sup>5</sup>

$$D_i^a = \alpha_i(Q_i^\dagger T^{a_i} Q_i - \tilde{Q}_i^\dagger T^{a_i} \tilde{Q}_i), \quad (2.12)$$

$$D_X = \sum_{i=1,2} \alpha_i \left( q_{Q_i} Q_i^\dagger Q_i + q_{\tilde{Q}_i} \tilde{Q}_i^\dagger \tilde{Q}_i \right) + \alpha_\chi q_\chi \chi^\dagger \chi - \alpha_\phi \phi^\dagger \phi - \frac{\delta}{2} \left[ \alpha'_i (Q_i^\dagger Q_i + \tilde{Q}_i^\dagger \tilde{Q}_i) + \alpha'_\phi \phi^\dagger \phi + \alpha'_\chi \chi^\dagger \chi + K'_M \right], \quad (2.13)$$

where  $T^{a_i}$  in eq. (2.12) are the generators of  $SU(N_i)$  and  $\prime$  in eq. (2.13) stands for a derivative with respect to  $U$ .

## 2.1 Looking for non-SUSY Minkowski minima

A direct analytical study of the minima of  $V$  is a formidable task. However, we will see that it is possible to find non supersymmetric metastable Minkowski minima starting from AdS SUSY vacua when  $\chi \ll 1$ .

Particularly important for what follows is the  $U(1)_X$  D-term. As well-known, the GS coefficient  $\delta$  induces a (field-dependent) FI term in  $D_X$ , as can be seen from eq. (2.13). The minimization of  $D_X$  induces then a non-vanishing VEV for  $\phi$  (taken real for simplicity):

$$\phi_{\text{SUSY}}^2 = \frac{-\delta K'_M}{2(\alpha_\phi + \delta\alpha'_\phi/2)}. \quad (2.14)$$

Notice that typically  $K'_M < 0$ , so that the right-hand side in eq. (2.14) is positive and the  $U(1)_X$  symmetry is spontaneously broken. From the third term in the superpotential (2.3), we see that  $\phi_{\text{SUSY}}$  also induces a mass term for the quarks  $Q_i$  and  $\tilde{Q}_i$ . Assuming that  $\phi_{\text{SUSY}} \gg m_{3/2}$ , unless the Yukawa couplings  $c_i(U, Z, \phi)$  are extremely small, a sufficiently large mass for the quarks  $Q_i$  and  $\tilde{Q}_i$  is induced.<sup>6</sup> Under the assumption that at the minimum  $W_h \ll 1$ , which is obviously required to have a sufficiently light gravitino mass, the quarks can be integrated out by safely neglecting all supergravity and moduli corrections, by setting to zero  $D_i$  and their flat-space F-terms:  $F_{M_i} = \partial W_h / \partial M_i = 0$ , where  $M_i = Q_i \tilde{Q}_i$  are the ‘‘meson’’ chiral fields. The effective superpotential  $W_{\text{eff}}$  which result after having integrated out the quark superfields reads

$$W_{\text{eff}} = w_0 + f(\phi) e^{-\gamma_Z Z - \gamma_U U} \chi + \sum_{i=1,2} A_i(\phi) e^{-a_i U - b_i Z}, \quad (2.15)$$

where

$$a_i \equiv \eta_{i,U} \frac{N_{if}}{N_i} + \frac{8\pi^2 n_i}{N_i}, \quad b_i \equiv \eta_{i,Z} \frac{N_{if}}{N_i} + \frac{8\pi^2 m_i}{N_i},$$

$$f(\phi) \equiv Y \phi^{\hat{q}_\chi}, \quad A_i(\phi) \equiv \eta_i N_i e^{-8\pi^2 p_i / N_i} \left( c_i \phi^{q_i} \right)^{\frac{N_{if}}{N_i}}, \quad (2.16)$$

<sup>5</sup>Below and throughout the paper, we use the same notation to denote a chiral superfield and its lowest scalar component, since it should be clear from the context to what we are referring to.

<sup>6</sup>We assume that the  $c_i$  in eq. (2.6) are such that all quarks get a mass when  $\phi$  acquires a VEV.



are effective parameters and for simplicity we have defined the effective charges

$$q_i \equiv q_{Q_i} + q_{\bar{Q}_i} + \eta_{i,U} \delta / 2, \quad \hat{q}_\chi \equiv q_\chi + \frac{\gamma_U \delta}{2}. \quad (2.17)$$

The meson VEV's are negligibly small, in agreement with our assumption:

$$\langle M_i \rangle = \Lambda_i^2 \left( \frac{\Lambda_i}{m_i} \right)^{1 - N_{if}/N_i}, \quad (2.18)$$

with  $m_i = c_i \phi^{q_i}$ , being both suppressed by the dynamically generated scales  $\Lambda_i \ll 1$  and the small ratio  $\Lambda_i/m_i$ .<sup>7</sup> Correspondingly, we can completely neglect the mesons in  $K_h$ , so that the resulting effective Kähler potential is simply

$$K_{\text{eff}} = K_M + \alpha_\phi \phi^\dagger e^{-2V_X} \phi + \alpha_\chi \chi^\dagger e^{2q_\chi V_X} \chi. \quad (2.19)$$

As next step, we look for vacua with  $\chi \ll 1$ . We expand the scalar potential  $V_{\text{eff}}$  arising from (2.15), (2.19) and the  $D_X$  term in powers of  $\chi$ ,  $V_{\text{eff}} = \sum_{n,m=0}^\infty V_{n,m} \chi^n \chi^{\dagger m}$ , and keep only the leading term  $V_0 \equiv V_{0,0}$ . It reads

$$V_0 = \frac{1}{2\text{Re} f_X} (D_X^{(0)})^2 + e^{K_{\text{eff}}^{(0)}} \left[ \sum_{i,j=U,Z,\phi} K_{\text{eff}}^{(0)i\bar{j}} F_i^{(0)} F_{\bar{j}}^{(0)} - 3|W_{\text{eff}}^{(0)}|^2 + V_{up-lift} \right], \quad (2.20)$$

where the  $F$ -terms are computed using  $K_{\text{eff}}$  and  $W_{\text{eff}}$  and the superscript (0) means that all expressions are evaluated for  $\chi = 0$ . The first three terms in  $V_0$  correspond to the SUGRA scalar potential that would result from  $K_{\text{eff}}^{(0)}$ ,  $W_{\text{eff}}^{(0)}$  and  $f_X$ . The last term

$$V_{up-lift} = \frac{|F_\chi^{(0)}|^2}{\alpha_\chi}, \quad (2.21)$$

where  $F_\chi^{(0)} = f(\phi) \exp(-\gamma_Z Z - \gamma_U U)$ , is effectively a moduli-dependent ‘‘up-lifting’’ term.

Let us look for approximate SUSY vacua for  $U$  and  $Z$ , neglecting for the moment the up-lifting term  $V_{up-lift}$ , and assuming that at the extremum  $w_0$  is larger than the dynamically generated terms in (2.15). It is easy to solve the system  $D_X^{(0)} = F_U^{(0)} = F_Z^{(0)} = F_\phi^{(0)} = 0$ . The  $D_X$  term trivially vanishes when eq. (2.14) is satisfied, so that  $\phi$  is determined. At a SUSY extremum, gauge invariance implies  $F_\phi^{(0)} = -\delta/(2\phi)F_U^{(0)}$ , so that we are left to solve the system  $F_U^{(0)} = F_Z^{(0)} = 0$ . We get

$$U_{\text{SUSY}} \simeq \frac{b_2 x_1 - b_1 x_2}{a_1 b_2 - a_2 b_1}, \quad Z_{\text{SUSY}} \simeq \frac{a_1 x_2 - a_2 x_1}{a_1 b_2 - a_2 b_1}, \quad (2.22)$$

where

$$x_{1,2} = -\log \left[ \pm \frac{w_0}{A_{1,2}(\phi_{\text{SUSY}})} \frac{b_{2,1} K_{\text{eff}}^{(0)'} - a_{2,1} \dot{K}_{\text{eff}}^{(0)}}{a_1 b_2 - a_2 b_1} \right], \quad (2.23)$$

and a dot stands for a derivative with respect to  $Z$ . By appropriately choosing the signs of  $\eta_i$  appearing in  $A_i(\phi)$ , see eq. (2.16), we can always set  $U_{\text{SUSY}}$  and  $Z_{\text{SUSY}}$  to be real, so

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<sup>7</sup>Of course, we are assuming at this stage the existence of a non-runaway minimum for the moduli  $U, Z$ .

that for simplicity of notation in the following we will always assume real fields and real parameters. Since  $U$  and  $Z$  enter explicitly in the coefficients  $x_{1,2}$  above, eqs. (2.22) do not admit explicit analytic solutions. However, the logarithmic dependence on  $U$  and  $Z$  of  $x_{1,2}$  is often mild enough that a good approximate expression for  $U_{\text{SUSY}}$  and  $Z_{\text{SUSY}}$  is obtained by taking some educated guess for the moduli in eq. (2.23), compute (2.22), insert the result in (2.23) and compute once again (2.22). The shifts in the fields due to the up-lifting term  $V_{up-lift}$  can be found by expanding the extrema conditions  $\partial_U V_0 = \partial_Z V_0 = \partial_\phi V_0 = 0$  around the SUSY values (2.22):  $U = U_{\text{SUSY}} + \Delta U$ ,  $Z = Z_{\text{SUSY}} + \Delta Z$ ,  $\phi = \phi_{\text{SUSY}} + \Delta\phi$  and keeping the leading term in  $V_{up-lift}$  and terms up to linear order in  $\Delta U$ ,  $\Delta Z$  or  $\Delta\phi$  in the remaining terms of the scalar potential. The resulting expressions one gets for the shifts are actually very involved and can be handled only numerically. Some simple approximate formulae can however be derived by using simple scaling arguments to estimate the typical size of the terms entering in the Kähler and superpotential terms, eqs. (2.15) and (2.19). We first notice that eq. (2.22) fixes the sizes of the moduli  $U$  and  $Z$  at the SUSY point to be inversely proportional to the effective parameters  $a_{1,2}$  and  $b_{1,2}$ . For simplicity, we can take all the  $a_{1,2}$  and  $b_{1,2}$  parameters that do not vanish to be of the same order of magnitude and denote their common value by  $a$ . It then follows that  $U \sim Z$  and we can generally denote by  $X$  the common modulus VEV. We will use such simplified notation anytime we want to estimate a quantity without giving its explicit expression. Coming back to eq. (2.22), it is clear that  $aX$  is approximately a constant, proportional to the  $x_{1,2}$  coefficients defined in eq. (2.23). Since  $w_0 \ll 1$  is required to have a sufficiently light gravitino mass, this constant is much larger than 1. As a matter of fact, for a wide class of models  $aX$  is always in the narrow range  $20 \lesssim aX \lesssim 40$ , which is essentially the range dictated by the Planck/electroweak scale hierarchy. The parameter  $\epsilon \equiv 1/(aX)$  is then small and an expansion in  $\epsilon$  is possible. The following scaling behaviours are taken at the SUSY extremum:

$$\begin{aligned} \partial_X^n K_{\text{eff}}^{(0)} &\sim \partial_X^n K_M \sim \frac{1}{X^n}, & \partial_X^n \alpha_{\phi,X} &\sim \frac{\alpha_{\phi,X}}{X^n}, & n > 0. & \quad (2.24) \\ \partial_X W_{\text{eff}}^{(0)} &\sim (\partial_X K_{\text{eff}}^{(0)}) W_{\text{eff}}^{(0)} \sim \frac{1}{X} W_{\text{eff}}^{(0)}, & \partial_X^n W_{\text{eff}}^{(0)} &\sim a^{n-1} \partial_X W_{\text{eff}}^{(0)}, & n > 1. & \end{aligned}$$

The term proportional to  $|\phi|^2$  in  $K_{\text{eff}}^{(0)}$  is sub-leading in  $\epsilon$  with respect to the purely moduli dependent term  $K_M$ . Indeed,  $\alpha_\phi |\phi|^2 \lesssim \delta/X$  and, neglecting possible corrections due to  $\eta_{iZ/U}$ , one has from eqs. (2.5) and (2.16) that  $\delta = 2q_i N_{if} / (N_i a_i)$ , which typically is less or equal to  $1/a$ . This explains the first relation in eq. (2.24). We are now ready to estimate the shift of the fields  $\Delta U$ ,  $\Delta Z$  and  $\Delta\phi$  due to the up-lifting term  $V_{up-lift}$ . In the heavy  $U(1)_X$  gauge field approximation  $M_X \gg m_{3/2}$ , which will always be the case of interest for us, one has (see e.g. [28])

$$\langle D_X \rangle \simeq \frac{2}{M_X^2} e^{K_{\text{eff}}^{(0)}} \frac{q_X (F_X^{(0)})^2}{\alpha_X} = \frac{2}{M_X^2} q_X e^{K_{\text{eff}}^{(0)}} V_{up-lift}, \quad (2.25)$$

where  $M_X^2 = 2g_X^2 \alpha_\phi \phi_{\text{SUSY}}^2$ , so that the  $D_X$  term at the minimum is negligibly small. We can use the condition  $D_X \simeq 0$  to express  $\Delta\phi$  as a function of  $\Delta Z$  and  $\Delta U$ . Using eq. (2.14),

it is straightforward to see that  $\Delta\phi$  scales as

$$\Delta\phi \sim \phi \frac{\Delta X}{X}. \quad (2.26)$$

Next step is to estimate  $\Delta X$ . Using eqs. (2.14), (2.24), (2.25) and (2.26), it is a simple exercise to see that at leading order in  $\epsilon$  and up to linear order in  $\Delta X$ , one has

$$\partial_X V_0 \simeq e^{K_M^{(0)}} K_M^{X\bar{X}} \partial_{\bar{X}} F_{\bar{X}} \partial_X F_X \Delta X + \partial_X (e^{K_M^{(0)}} V_{up-lift}) + e^{K_M^{(0)}} q_X V_{up-lift} \frac{\partial_X^2 K_M}{\partial_X K_M} \simeq 0, \quad (2.27)$$

giving

$$\Delta X \sim V_{up-lift} \frac{\partial_X^2 K_M \left[ q_X \partial_X^2 K_M + (\partial_X K_M)^2 + \gamma \partial_X K_M \right]}{\partial_X K_M \left( \partial_X^2 W_{\text{eff}}^{(0)} \right)^2} \sim \epsilon^2 \frac{V_{up-lift}}{(W_{\text{eff}}^{(0)})^2} X (q_X + \gamma X), \quad (2.28)$$

where  $\gamma$  generally denotes  $\gamma_U$  or  $\gamma_Z$  and we have tacitly assumed  $q_X > 1$  in writing the last relation in eq. (2.28). At a Minkowski minimum,  $|V|_{up-lift} \sim |W_{\text{eff}}^{(0)}|^2$ , so that the fraction in eq. (2.28) is  $O(1)$ . If  $\gamma_U = \gamma_Z = 0$  and  $q_X \sim O(1)$ , the relative shifts of the moduli are small:  $\Delta X/X \sim \epsilon^2 \sim 10^{-3}$  and certainly the up-lifting term  $V_{up-lift}$  does not de-stabilize the system and can be treated as a perturbation, as we did. When  $\gamma_U$  and/or  $\gamma_Z$  are non-vanishing, the up-lifting term  $V_{up-lift}$  becomes exponentially sensitive to the values of  $U, Z$ . It is then not enough to have  $\Delta X/X \ll 1$ , but the stronger constraint  $\gamma \Delta X \ll 1$  is required, in order to avoid large displacements of  $V_{up-lift}$  which can result on the impossibility of finding a Minkowski solution. This results on a bound on the size of  $\gamma$ :

$$\gamma^2 \ll a^2. \quad (2.29)$$

The same constraint  $\gamma \Delta X \ll 1$  gives also an upper bound on  $q_X$ ,  $\epsilon q_X \gamma / a \ll 1$ , which is however quite mild, in light also of eq. (2.29). We do not report the detailed expressions for  $\Delta U$  and  $\Delta Z$  in the general case, which are very involved and not illuminating even when expanded in powers of  $\epsilon$ . Just for concreteness, we report their form for the particular case when  $\alpha_\chi = \alpha_\phi = 1$ , factorizable  $K_M, W_{\text{eff}}$  (i.e.  $\dot{K}'_M = \dot{W}'_{\text{eff}} = 0$ ) and perturbatively generated  $Y$  Yukawa coupling:  $\gamma_U = \gamma_Z = 0$ . In these approximations, we have

$$\begin{aligned} \Delta U &\simeq -V_{up-lift} \frac{K_M'' [(K'_M)^2 + q_X K_M'']}{K'_M (W_{\text{eff}}^{(0)''})^2}, \\ \Delta Z &\simeq -V_{up-lift} \frac{\dot{K}_M \ddot{K}_M}{(\ddot{W}_{\text{eff}}^{(0)})^2}, \end{aligned} \quad (2.30)$$

whose scalings are in agreement with the general estimate (2.28). Notice that both  $\Delta Z$  and  $\Delta U$  are positive, since  $\dot{K}_M$  and  $K'_M$  are negative, tending to decrease the up-lifting term. The scaling behaviours of the  $F$  terms at the non-SUSY vacuum are easily found:

$$\begin{aligned} F_\chi^{(0)} &\sim \alpha_\chi^{1/2} W_{\text{eff}}^{(0)}, \\ F_X^{(0)} &\sim \epsilon \left( \gamma + \frac{q_X}{X} \right) \alpha_\chi^{-1/2} F_\chi^{(0)}, \\ F_\phi^{(0)} &\sim \frac{\delta}{\phi} \epsilon \left( \gamma + \frac{q_X}{X} \right) \alpha_\chi^{-1/2} F_\chi^{(0)}. \end{aligned} \quad (2.31)$$

Using eq. (2.31), one can easily estimate  $K_{\text{eff}}^{(0)i\bar{j}} F_i^{(0)} F_{\bar{j}}^{(0)}$  and  $V_{up-lift}$ . In agreement with our expectation,  $V_{up-lift}$  is the leading term contributing positively to the vacuum energy, justifying its name of up-lifting term. It is also possible to verify the validity of our approximation of having integrated out the meson fields  $M_i$ . Having integrated them out in the flat-space limit, the  $F_{M_i}$  at the non-SUSY vacuum scale as in [29] and are of order  $F_{M_i} \sim W_{\text{eff}}^{(0)}$ . However, they are totally negligible due to powers of  $M_i$  which appear in the Kähler metric, i.e.  $F^{M_i} \sim M_i F_{M_i} \ll F_{M_i}$ .<sup>8</sup>

Once the approximate vacuum of the leading potential  $V_0$  has been found, given by  $U_0 = U_{\text{SUSY}} + \Delta U$ ,  $Z_0 = Z_{\text{SUSY}} + \Delta Z$ ,  $\phi_0 = \phi_{\text{SUSY}} + \Delta\phi$ , we turn on  $\chi$  and verify the validity of our initial assumption  $\chi \ll 1$ . The easiest way to estimate  $\chi$  is to use the second relation in eq. (2.11) that relates the  $D_X$  term to the  $F$ -terms. At the minimum  $D_X \simeq 0$ , but the  $F$ -terms for  $\chi$  and  $\phi$  contributing to  $D_X$  are typically of the same order of magnitude:  $\phi F_\phi \sim q_\chi \chi F_\chi$ . Using this relation and the scalings (2.31), we get

$$\chi_0 \sim \delta\epsilon \left( \gamma + \frac{q_\chi}{X} \right) \frac{1}{q_\chi} \alpha_\chi^{-1/2}, \tag{2.32}$$

which proves our initial assumption  $\chi \ll 1$ . A more accurate estimate of  $\chi_0$  might be obtained by considering the next sub-leading potential terms  $V_{1,0} = V_{0,1}$ ,  $V_{2,0} = V_{0,2}$  and  $V_{1,1}$  obtained by expanding  $V_{\text{eff}}$  in powers of  $\chi$  and  $\chi^\dagger$ . In first approximation, one can freeze  $U$ ,  $Z$  and  $\phi$  at the values  $U_0$ ,  $Z_0$  and  $\phi_0$ , which extremize  $V_0$ , so that  $\chi$  is determined by a linear equation. The general explicit expression for  $\chi$  is however very involved and not very interesting, so we will not report it here. For the same reason, we do not report the expressions of the further shifts of  $U$ ,  $Z$  and  $\phi$  induced by the backreaction of  $\chi$ . They turn out to scale as eqs. (2.26) and (2.28), but are typically smaller in the parameter region we will consider in the following. The location of the vacuum is then slightly shifted but it is not destabilized by the field  $\chi$ . We can also check how  $\chi$  changes the values of the  $F$ -terms (2.31). One has

$$\begin{aligned} F_\chi &= F_\chi^{(0)} + \alpha_\chi \chi \left( W_{\text{eff}}^{(0)} + \chi F_\chi^{(0)} \right), \\ F_X &= F_X^{(0)} + \chi \left( \partial_X F_\chi^{(0)} + \partial_X K_{\text{eff}} F_\chi^{(0)} \right), \\ F_\phi &= F_\phi^{(0)} + \chi \left( \frac{\hat{q}_\chi}{\phi} + \phi \alpha_\phi \right) F_\chi^{(0)}. \end{aligned} \tag{2.33}$$

Using eqs. (2.31) and (2.32), it is straightforward to verify that the effect of  $\chi$  on  $F_\chi$  and  $F_X$  is negligible, while the second term in  $F_\phi$  is of the same order as  $F_\phi^{(0)}$ . Hence the scalings (2.31) hold also for the full  $F$ -terms  $F_\chi$ ,  $F_X$  and  $F_\phi$ , providing a final consistency check of eq. (2.32), which has been derived under this assumption.

Let us now discuss under which conditions the above SUSY breaking mechanism is stable under small deformations. The choice of the superpotential (2.3) was rather ad hoc, since we have considered only linear terms in  $\chi$  and tacitly assumed that possible

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<sup>8</sup>The meson masses induced by the VEV's of  $\phi$  and the moduli are typically much higher than the dynamically generated scales, so that one can safely take the classical Kähler potential for the mesons:  $K(M_i) = 2\text{Tr}(M_i^\dagger M_i)^{1/2}$ . Equivalently, the description in terms of quarks is valid at these energy scales.

higher order terms of the form  $Y_n(U, Z, \phi)(\chi\phi^{q_\chi})^n$ , with  $n > 1$ , can be neglected. This assumption is actually very strong, since the requirement  $|F_\chi^{(0)}| \simeq \alpha_\chi^{1/2}|W_{\text{eff}}^{(0)}|$  puts severe constraints on the size of the constant term  $Y$  appearing in (2.6). This is particularly clear if one notices that generally  $\alpha_\chi \lesssim 1$  and  $W_{\text{eff}}^{(0)} \simeq w_0 \lesssim O(10^{-13})$ . The more obvious options of assuming a perturbative ( $\gamma_{U,Z} = 0$ ) mass term ( $q_\chi = 1$ ) or trilinear coupling ( $q_\chi = 2$ ) leads to an unnaturally small coupling  $Y$ . In such a situation, the terms of the form  $Y_n(U, Z, \phi)(\chi\phi^{q_\chi})^n$  will lead to a restoration of SUSY and to the destabilization of the non-SUSY vacuum. This is best seen by considering the flat space model with stabilized moduli. In this case, the relevant superpotential term in eq. (2.15) is just the term linear in  $\chi$ , which is invariant under a  $U(1)_R$  symmetry with  $R(\chi) = 2$ ,  $R(\phi) = 0$ . An exact  $R$ -symmetry is generally necessary to get a SUSY-breaking vacuum [30] and, indeed, in absence of moduli and gravitational dynamics,  $\chi$  is stabilized at the origin where  $U(1)_R$  is unbroken. Any term of higher order in  $\chi$  will necessarily break  $U(1)_R$ , leading to the appearance of SUSY vacua. Gravity and moduli explicitly break  $U(1)_R$ , but if the breaking is small enough their only effect would be to displace a bit  $\chi$  from the origin, so that  $\chi \ll 1$ , as predicted by eq. (2.32). If the terms  $Y_n(U, Z, \phi)(\chi\phi^{q_\chi})^n$  are all negligibly small, much smaller than  $\chi w_0$ , the SUSY preserving vacua will appear for large values of  $\chi$  and will not perturb much the (meta)stable non-SUSY vacuum close to the origin. However, when the terms  $Y_n(U, Z, \phi)(\chi\phi^{q_\chi})^n$  become roughly of the same order as  $\chi w_0$ , the SUSY vacua approach the origin and the non-SUSY vacua are destabilized and disappear.

Invoking non-perturbatively generated couplings ( $\gamma \neq 0$ ) alleviate the problem, but it does not solve it, because a natural  $Y$  would require  $\gamma \sim a$ , so that

$$e^{-\gamma U} \sim e^{-aU} \sim W_{\text{eff}}^{(0)}, \tag{2.34}$$

but the constraint (2.29) does not allow such values of  $\gamma$ . A possible way out is to consider higher-order couplings by taking  $q_\chi > 2$ , so that the effective term  $\phi_0^{q_\chi}$  becomes small enough to get not so small values of  $Y$ . In this way, we also more effectively suppress the dangerous terms  $(\chi\phi^{q_\chi})^n$ . Summarizing, the requirement of naturalness and more importantly stability under superpotential deformations with higher powers of  $\chi$  necessarily require to consider non-renormalizable interactions with  $q_\chi > 2$ , the precise bound on  $q_\chi$  depending on  $\gamma_{U,Z}$  being zero or not.<sup>9</sup>

So far, we have been able to find approximate expressions for the extrema of the scalar potential  $V_{\text{eff}}$ , but we have still to check whether these vacua are minima or not. At leading order in  $\epsilon$  and for  $\phi_{\text{SUSY}}/X \ll 1$ , the kinetic mixing of  $\phi$  with the moduli can be neglected and the mass of  $\phi$  is determined by the D-term potential. Its physical mass is

$$m_\phi^2 \simeq 2g_X^2 \alpha_\phi \phi_{\text{SUSY}}^2, \tag{2.35}$$

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<sup>9</sup>A cheap way to overcome this problem is to assume that  $Y$  is an effective non-perturbatively generated coupling of some other modulus which has been already stabilized. This simple explanation should be avoided, because we have tacitly assumed that all possible other moduli have been stabilized supersymmetrically. This implies that an effective decoupling has to occur between them and the sector of the theory responsible for SUSY breaking. This assumption is manifestly violated if one considers the above cheap explanation.

which is also the mass of the gauge vector boson  $A_X$ , as we have seen. Indeed,  $\text{Im } \phi$  is approximately the would-be Goldstone boson eaten up by  $A_X$  after the  $U(1)_X$  gauge symmetry breaking. The leading contribution to the  $\chi$  mass is also easily derived by looking at the terms quadratic in  $\chi, \chi^\dagger$ . Its physical mass equals

$$m_\chi^2 \simeq \frac{2q_\chi^2}{\alpha_\phi \phi_{\text{SUSY}}^2} V_{up-lift}. \tag{2.36}$$

From eq. (2.36) we have  $m_\chi \gtrsim (q_\chi/\sqrt{\epsilon}) m_{3/2} \gg m_{3/2}$  and hence the effects of SUSY breaking on  $m_\chi$  are small, so that  $m_\chi$  is approximately the mass of both components of the complex field  $\chi$ . The mass scale of  $m_\phi$  is of order  $\sqrt{\epsilon/\bar{X}}$  and, unless the moduli are very large, it is just one or two orders of magnitude below the Planck scale. The moduli masses are more involved and are best described by an effective approach where the massive fields  $\phi$  and  $A_X$  are integrated out, approach we will now consider.

## 2.2 Effective description

As we have seen, the cancellation of the  $U(1)_X$  D-term given by a non-trivial VEV for  $\phi$  induces large meson masses, as well as large masses for  $A_X$  and  $\phi$  itself. One might then not only integrate out the meson fields, as we did, but also  $A_X$  and  $\phi$ . In first approximation, we can integrate out  $\phi$  and  $A_X$  at the SUSY level. A useful way to do that is to go to the super-unitary gauge, which is nothing else than the super-field version of the standard unitary gauge. In our case the super-unitary gauge reads

$$\phi = \phi_0 + \frac{\delta K_M''}{2\phi_0 \alpha_\phi} (U - U_0) + \frac{\delta \hat{K}'_M}{2\phi_0 \alpha_\phi} (Z - Z_0), \tag{2.37}$$

where  $\phi_0, U_0$  and  $Z_0$  are the approximate VEV's we have previously found. The would-be Goldstone boson is essentially given by  $\text{Im } \phi$ , being the last two terms in eq. (2.37) suppressed at least by a factor  $\sqrt{\epsilon}$ . In this way, we can get rid of the  $\phi$  chiral field, substituting eq. (2.37) (and its anti-chiral version) in both the Kähler potential (2.19) and superpotential (2.15). At leading order, we can neglect the last two terms in eq. (2.37) and just take  $\phi = \phi_0$ . We can then expand the resulting effective Kähler potential  $K_{\text{eff}}(\phi_0)$  for small  $V_X$  and keep up to quadratic terms in  $V_X$ . Then the equation of motion for  $V_X$ ,  $\partial K_{\text{eff}}(\phi_0)/\partial V_X = 0$  is easily solved and one finds

$$V_X \simeq -\frac{D_X}{2\alpha_\phi \phi_0^2} \simeq -\frac{q_\chi \alpha_\chi}{2\alpha_\phi \phi_0^2} |\chi|^2, \tag{2.38}$$

where in the last relation we have completely neglected the  $U$  and  $Z$  dynamics by taking  $D_X \simeq q_\chi \alpha_\chi |\chi|^2$ . By plugging back eq. (2.38) in  $K_{\text{eff}}(\phi_0)$ , we get the following effective Kähler potential at leading order [31]:

$$\hat{K}_{\text{eff}} \simeq \alpha_\chi |\chi|^2 + K_M - \frac{D_X^2}{2\alpha_\phi \phi_0^2} \simeq \alpha_\chi |\chi|^2 + K_M - \frac{q_\chi^2 \alpha_\chi^2}{2\alpha_\phi \phi_0^2} |\chi|^4. \tag{2.39}$$

The superpotential  $\hat{W}_{\text{eff}}$  trivially follows from (2.15) with  $\phi = \phi_0$ , so that the field dependent terms  $f(\phi)$  and  $A(\phi)$  defined in eq. (2.16) become now effective constants  $f(\phi_0)$  and

$A(\phi_0)$ . Notice that  $\hat{K}_{\text{eff}}$  and  $\hat{W}_{\text{eff}}$  sensitively depend on  $\phi_0$ , whose precise value cannot be correctly determined without actually using the full underlying model. However, the shifts on  $\phi$ , as computed in the previous subsection, are small enough that at leading order one might safely replace  $\phi_0$  in all the above formulae (and the one that will follow) by the SUSY value (2.14).

The effective model described by  $\hat{K}_{\text{eff}}$  and  $\hat{W}_{\text{eff}}$  is considerably more tractable than the underlying UV model we considered before. In particular, some physical features are more transparent and, in addition, such effective description provides us with an approximate formula for the moduli masses. For instance, it is immediately clear that a vacuum with  $\chi_0 \ll 1$  and non-runaway moduli will necessarily break SUSY, since  $F_\chi \simeq f \exp(-\gamma_Z Z_0 - \gamma_U U_0) \neq 0$ . In fact, as far as  $\chi$  is concerned, the model is nothing else than a Polonyi model with a deformed Kähler potential. As we already mentioned, in the flat-space limit with decoupled moduli,  $\chi$  will be stabilized at the origin due to the  $|\chi|^4$  term in eq. (2.39). The extrema conditions (2.22) for  $U$  and  $Z$  are rederived by requiring  $\hat{F}_U^{(0)} = \hat{F}_Z^{(0)} = 0$ , using the same notation as before. An approximate analytical formula for the moduli mass terms can be derived, once again by keeping the leading term in an expansion in  $\epsilon$ . One gets

$$m_{i\bar{j}}^2 \simeq e^{K_M} K_M^{\bar{m}l} \partial_i \partial_l \hat{W}_{\text{eff}}^{(0)} \partial_{\bar{j}} \partial_{\bar{m}} \bar{\hat{W}}_{\text{eff}}^{(0)}, \tag{2.40}$$

with the indices running over  $U$  and  $Z$ . Since  $K_M$  is a function of the real part of the moduli only, we can drop any distinction between holomorphic and anti-holomorphic indices in taking derivatives of the Kähler potential  $K_M$ . The Kähler metric  $g$  for the moduli is then a real symmetric matrix which is diagonalized by an orthogonal  $\text{SO}(2)$  matrix  $C$ :  $C^t g C = d$ , with  $d$  a diagonal matrix. It follows that the physical moduli masses are given by the eigenvalues of the following mass matrix:

$$d^{-1/2} C m^2 C^t d^{-1/2}. \tag{2.41}$$

In order to have a minimum, we have to require that the mass matrix (2.41) is positive definite. This will in general give a non-trivial constraint on the possible form of  $K_M$  and the parameter space of the model. However, such a constraint is not very restrictive, as can be seen by considering, for instance, the particular case of decoupled Kähler and superpotential terms, namely  $\hat{K}'_M = \hat{W}'_{\text{eff}} = 0$ . In this simple case, both the moduli masses and kinetic terms are already in a diagonal form, and we get

$$m_{U,Z}^2 \simeq e^{K_M} \left| \frac{\partial_{U,Z}^2 \hat{W}_{\text{eff}}^{(0)}}{\partial_{U,Z}^2 K_M} \right|^2 \sim \epsilon^{-2} m_{3/2}^2, \tag{2.42}$$

which is manifestly positive definite. The last equality of eq. (2.42) shows the scaling of the moduli masses with respect to the gravitino mass  $m_{3/2}$ . As can be seen, the moduli are parametrically heavier than the gravitino, which is a cosmologically welcome feature.

An important comment is now in order. One might wonder why we have decided to adopt from the beginning an effective field theory approach for the hidden mesons (and tacitly for all the other moduli possibly responsible for the constant term in the

superpotential), integrating them out from the very beginning, and not for  $\phi$  and  $A_X$  which are actually even heavier ! From a purely effective quantum field theory point of view, this is indeed not justified, the correct procedure being the integration of all the states in the order specified by their mass scales and run the effective parameters down to lower energies. At the classical level we are considering here and when focusing only on the properties of the vacua, however, no real difference occurs and integrating out some state or not is only a matter of simplicity. Contrary to the mesons, which can always be easily integrated out supersymmetrically to a very good approximation, as we did,  $\phi$  and  $A_X$  would require more work than what we have shown above to be properly integrated out, because they are more sensible to SUSY breaking effects. If one wants to go beyond the leading terms in  $\epsilon$  and study more quantitatively the model, in particular, we should keep the full super-unitary gauge (2.37) instead of taking  $\phi = \phi_0$ . More importantly, the approximation of completely neglecting the moduli dynamics in  $D_X$  and substitute it with just  $q_\chi \alpha_\chi |\chi|^2$ , as we did in eqs. (2.38) and (2.39), turns out to be in general a quite crude approximation. Both these approximations can be relaxed and we have analytically and numerically checked that the resulting “improved” effective model reproduce pretty well, at a more quantitative level, the main properties of the full theory. Contrary to the naive effective theory we have shown above, however, the improved theory is no less complicated than the full one, so that no real simplification occurs in considering it. On the contrary, the naive model captures all the qualitative features of the full model and, as a matter of fact, it has been crucial to guide us in the analysis of the previous subsection.

The analysis of the general two-moduli model performed so far can trivially be reduced to the single modulus ( $U$ ) case with essentially no effort when  $w_0 \neq 0$ . We will then not repeat the analysis here, but just point out that all the considerations we made for the two-moduli case apply. The only qualitative difference is that with a single modulus one condensing gauge group is enough to get (meta)stable vacua. In particular, we have reproduced within our approach the results of [19] for a KKLT-like model with a single modulus.

### 3. Models with $w_0 = 0$

From the previous discussion it is clear that in general one has to face a possible fine-tuning problem to get  $Y \ll 1$  in the Yukawa sector for  $\chi$ , but one might argue that this is, by means of the flatness condition, just a reflection of the other fine-tuning problem required to have a tiny constant superpotential term,  $w_0 \ll 1$ . In principle, then, we have to face up to three fine-tuning problems, the third being the unavoidable tuning of the cosmological constant. Moreover, up to possible suppressions coming from the  $\exp(K)$  term in the scalar potential,  $w_0$  essentially fixes the supersymmetry breaking scale. It would be more desirable, instead, to dynamically generate it. This motivates us to analyze also the case in which  $w_0$  vanishes, assuming that some stringy symmetry forbids its appearance. Let us start by considering a theory with a single modulus ( $U$ ). Most of the considerations we made for  $w_0 \neq 0$  continue to apply for  $w_0 = 0$ , the main difference being the moduli stabilization mechanism, which now boils down to a racetrack model [22], where the scale of supersymmetry breaking is dynamically generated. The effective Kähler potential is still



given by eq. (2.19), with the obvious understanding that  $K_M$ ,  $\alpha_\phi$  and  $\alpha_\chi$  do not depend on  $Z$ . Similarly, the effective superpotential is as in eq. (2.15), with  $\gamma_Z = b_1 = b_2 = w_0 = 0$ . The condition of vanishing  $D_X$  term still fixes  $\phi$  to the value (2.14). The equation  $F_U^{(0)} = 0$ , at leading order in  $\epsilon$  has as solution

$$U_{\text{SUSY}} = \frac{1}{a_1 - a_2} \ln \left( -\frac{a_1 A_1(\phi_{\text{SUSY}})}{a_2 A_2(\phi_{\text{SUSY}})} \right). \quad (3.1)$$

The axionic component of  $U$  is always extremized such that the two condensing sectors get opposite signs, therefore for simplicity we take  $\eta_1 = -\eta_2 = 1$  and set it to zero. The scaling relations reported in eq. (2.24) are still valid but the very last relation among the derivatives of the superpotential should be reviewed. Indeed, the racetrack models work using the competing effects of the different condensing sectors, as clearly illustrated by eq. (3.1). As a result, there are some cancellations among the condensing scales that hold at the  $F$ -term level, but are destroyed once derivatives of  $F$ -terms are taken. The result of this is that the scaling behavior of the first derivative of the superpotential still satisfies the relation given by (2.24), but the higher ones are changed to

$$\partial_U^n W_{\text{eff}}^{(0)} \sim a^n W_{\text{eff}}^{(0)}, \quad n > 1. \quad (3.2)$$

Eq. (2.27) and the first relation in eq. (2.28) still hold with obvious notation changes, so using eq. (3.2), we now get for the modulus shift:

$$\Delta U \sim \epsilon^4 \frac{V_{\text{up-lift}}}{(W_{\text{eff}}^{(0)})^2} U(q_\chi + \gamma U), \quad (3.3)$$

where we omit the unnecessary subscript  $U$  in  $\gamma$ . Comparing eq. (3.3) with eq. (2.28), we notice that the shift of  $U$  induced by the up-lifting term is now two or three orders of magnitude smaller than the shifts in the models with  $w_0 \neq 0$ , being  $O(\epsilon^4)$  instead of  $O(\epsilon^2)$ . The  $F$ -terms are also parametrically smaller than before:

$$F_U^{(0)} \sim \epsilon^2 \left( \gamma + \frac{q_\chi}{U} \right) \alpha_\chi^{-1/2} F_\chi^{(0)}, \quad F_\phi^{(0)} \sim \frac{\delta}{\phi} \epsilon^2 \left( \gamma + \frac{q_\chi}{U} \right) \alpha_\chi^{-1/2} F_\chi^{(0)}, \quad (3.4)$$

where  $F_\chi^{(0)} \sim \alpha_\chi^{1/2} W_{\text{eff}}^{(0)}$ . The constraint  $\gamma \Delta U \ll 1$  gives now

$$\gamma^2 \ll \frac{a^2}{\epsilon^2}. \quad (3.5)$$

We see from eq. (3.5) that values of  $\gamma \sim a$  are now allowed, solving the fine-tuning problem in the coupling  $Y$ . The shift on the vacuum induced by the backreaction of  $\chi$  is now comparable or even slightly larger than eq. (3.3). However, it is typically small enough not to destabilize the vacuum. The scaling of  $\chi$  can still be estimated by the relation  $\phi F_\phi \sim q_\chi \chi F_\chi$ , giving

$$\chi_0 \sim \delta \epsilon^2 \left( \gamma + \frac{q_\chi}{X} \right) \frac{1}{q_\chi} \alpha_\chi^{-1/2}, \quad (3.6)$$

which is small, as required. The scalings of the full  $F$  terms (2.33) can easily be worked out using eq. (3.6). We find that  $F_\chi \sim F_\chi^{(0)}$ ,  $F_X \sim F_X^{(0)}$  and  $F_\phi \sim F_\phi^{(0)}$ , but the  $\chi$ -dependent

terms in  $F_X$  and  $F_\phi$  are non-negligible. The mass of  $U$  can be estimated using an effective description, as explained in subsection 2.2. The first relation in eq. (2.42) still holds, but the different scaling (3.2) of the superpotential gives now

$$m_U^2 \sim \epsilon^{-4} m_{3/2}^2. \quad (3.7)$$

The above analysis can be extended to the case of two moduli, in which case one has to work with at least three condensing sectors to get viable SUSY solutions. Instead of considering the most general model with three condensing gauge groups, we will now focus on an interesting class of models with decoupled non-perturbative superpotential terms. The effective superpotential  $W_{RT3}$  reads now

$$W_{RT3} = f(\phi)e^{-\gamma_Z Z - \gamma_U U} \chi + A_1(\phi)e^{-a_1 U} - A_2(\phi)e^{-a_2 U} + A_3(\phi)e^{-bZ}. \quad (3.8)$$

If the condensing scales associated to the gauge groups  $G_1$  and  $G_2$  are much larger than that of  $G_3$ ,  $U$  is approximately stabilized by a racetrack mechanism given by  $G_1$  and  $G_2$  at the value (3.1). With  $U$  so stabilized,  $W_{RT3}^{(0)}$  (notation as before) boils down to a KKLT-like superpotential which gives the following SUSY extremum for  $Z$  (see e.g. [32] for a similar analysis):

$$Z_{\text{SUSY}} \simeq -\frac{1}{b} \log \left[ \frac{\dot{K}_M \hat{w}_0}{b A_3} \right], \quad (3.9)$$

where

$$\hat{w}_0 \equiv A_1(\phi_{\text{SUSY}})e^{-a_1 U_{\text{SUSY}}} - A_2(\phi_{\text{SUSY}})e^{-a_2 U_{\text{SUSY}}} \quad (3.10)$$

is an effective constant superpotential term. The shifts in the fields induced by the up-lifting term  $V_{up-lift}$  can be derived using the by now familiar expansion in  $\epsilon \simeq 1/(bZ) \simeq 1/(a_1 U) \simeq 1/(a_2 U) \sim 1/(aU)$ . The form of the superpotential and the corresponding different stabilization mechanisms for  $Z$  and  $U$  do not allow now to consider  $U$  and  $Z$  together. Indeed, we have now  $\partial_U^n W_{RT3}^{(0)} \sim a^n W_{RT3}^{(0)}$ , as in eq. (3.2), and  $\partial_Z^n W_{RT3}^{(0)} \sim b^{n-1} W_{RT3}^{(0)}/Z$ , as in eq. (2.24). The leading terms in the shifts of the fields are as follows:

$$\Delta Z \sim \epsilon^2 q_\chi X + \epsilon^2 X^2 \gamma_Z + \epsilon^3 X^2 \gamma_U, \quad \Delta U \sim \epsilon^3 q_\chi X + \epsilon^3 X^2 \gamma_Z + \epsilon^4 X^2 \gamma_U, \quad (3.11)$$

where  $X$  denotes generically  $U$  or  $Z$ , assumed to be of the same order of magnitude. The usual bound  $\gamma_Z \Delta Z \ll 1$  does not allow for natural values  $\gamma_Z \sim a \sim b$ , so that we are forced to consider  $\gamma_Z = 0$  and  $\gamma_U \neq 0$ , in which case  $\gamma_U \sim a \sim b$  can be taken. The shift of  $\phi$  is given by  $\Delta \phi \sim \phi \Delta Z / Z$ . The  $F^{(0)}$ -terms scale as in eq. (3.4), with  $F_Z^{(0)} \sim F_U^{(0)}$ . The considerations made for  $\chi_0$  and the full  $F$ -terms in the single modulus case apply also here. The moduli masses depend on the form of the Kähler potential  $K_M$ . If  $K_M$  is factorizable, then  $U$  and  $Z$  will have masses roughly given by eqs. (3.7) and (2.42), respectively. If  $K_M$  is not factorizable, then generally the mass of  $U$  will be essentially as given by eq. (3.7), whereas  $Z$  will be heavier than what predicted by eq. (2.42), depending on the mixing between the two moduli in  $K_M$ .

#### 4. Soft masses

The natural framework of SUSY breaking mediation in any model with hidden and not sequestered sector, is gravity mediation, with  $m_{3/2} \sim O(\text{TeV})$ . A sufficiently heavy gravitino is desirable for cosmological reasons, being the moduli masses proportional to  $m_{3/2}$ , according to eqs. (2.42) and (3.7).<sup>10</sup> When the would-be anomalous  $U(1)_X$  gauge field is very massive, like the scenario advocated in our paper, one can effectively integrate  $A_X$  out and get the effective Kähler potential (2.39), taking care of including possible visible sector contributions to the  $D_X$  term, which we will shortly discuss. In this way, all soft parameters can be derived using the standard results of [34] with  $F$ -term breaking only. We will not explicitly compute all the resulting soft terms, but rather we will just estimate the size of the gaugino and scalar masses.

Let us start by considering the non-holomorphic soft scalar masses, in which case our considerations will apply to both the models with  $w_0 \neq 0$  and  $w_0 = 0$ . For canonically normalized fields, they read [34]:

$$m_{i\bar{i}}^2 = m_{3/2}^2 - \frac{1}{\alpha_{iv}} F^I F^{\bar{J}} R_{i\bar{i}I\bar{J}}. \quad (4.1)$$

In eq. (4.1),  $F^I = \exp(\hat{K}/2)\hat{K}^{I\bar{J}}F_{\bar{J}}$ ,  $I, J$  run over the hidden sector fields ( $U, Z$  and  $\chi$ ) and  $\alpha_{iv}$  is the moduli-dependent function appearing in the Kähler potential of the visible sector, eq. (2.1), with  $i$  running over the visible sector fields. Two possibilities arise, depending on whether the  $U(1)_X$  charges  $q_{iv}$  are vanishing or not. When  $q_{iv} \neq 0$ ,  $D_X \simeq \alpha_\chi q_\chi |\chi|^2 + \alpha_{iv} q_{iv} |Q^{(iv)}|^2$  and the leading canonically normalized soft mass terms reads

$$m_{i\bar{j}}^2 \simeq \delta_{i\bar{j}} m_{3/2}^2 \frac{3q_\chi q_{iv}}{\alpha_\phi |\phi_0|^2} \gtrsim \delta_{i\bar{j}} m_{3/2}^2 \frac{q_\chi q_{iv}}{\epsilon}, \quad (4.2)$$

which arise from the term  $F^X F^{\bar{X}} R_{\chi\bar{\chi}i\bar{j}}$  in eq. (4.1). Using eq. (2.25), eq. (4.2) can be also rewritten in the more conventional form  $m^2 = q_{iv} g_X^2 \langle D_X \rangle$ .

If  $q_{iv} = 0$ ,  $D_X \simeq \alpha_\chi q_\chi |\chi|^2 - \delta/2 \alpha'_{iv} q_{iv} |Q^{(iv)}|^2$ . The leading term coming from  $D_X$  is now of the same order as the universal  $m_{3/2}^2$  term appearing in eq. (4.1), so that

$$m_{i\bar{j}}^2 \sim \delta_{i\bar{j}} m_{3/2}^2. \quad (4.3)$$

In eq. (4.3) we have not considered the contribution of possible quartic terms in the charged fields of the form  $|Q_{iv}|^2 |\chi|^2$  which we have not specified in the Kähler potentials (2.1) and (2.2). Their contribution can be relevant or even dominant, but it is model-dependent and can easily be derived from eq. (4.1) once these terms are specified. Given eqs. (4.2) and (4.3), the choice  $q_{iv} = 0$  is preferred, giving rise to not too heavy scalar masses.

Let us now consider the gauginos. Their canonically normalized masses are

$$m_g = \left| F^I \frac{\partial_I f_v}{2\text{Re } f_v} \right|, \quad (4.4)$$

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<sup>10</sup>One might further push  $m_{3/2}$  to  $O(10 \text{ TeV})$  or more, assuming a sequestering of the hidden sector from the visible sector, so that the gravity mediation can be suppressed and anomaly mediation takes over [33]. We will not consider this possibility, which is non-generic.

where  $f_v$  schematically denotes the holomorphic gauge kinetic functions of the visible gauge group. Let us first discuss the models with  $w_0 \neq 0$ . In this case, using eq. (2.31), we can easily estimate, for linearly moduli dependent  $f_v$ ,

$$m_g \sim m_{3/2} \epsilon X \left( \gamma + \frac{q_X}{X} \right). \tag{4.5}$$

We see that  $m_g < m_{3/2}$ , but on the other hand eq. (4.5) predicts gaugino masses which are typically larger than those found in the original KKLT scenario with  $\bar{D}_3$  brane(s). This was already observed in [19] for a model with one modulus and perturbative up-lifting term ( $\gamma = 0$ ). We notice here that when  $\gamma \neq 0$  (or  $q_X > 1$ ), eq. (4.5) predicts even larger gaugino masses. In fact, considering that  $\epsilon X \gamma \sim \gamma/a$  and the bound (2.29), the gauginos can be made just a few times lighter than  $m_{3/2}$ , sufficiently heavy to neglect anomaly mediation contributions which become relevant if  $m_g \lesssim m_{3/2}/(4\pi)$ . We believe this is an important welcome feature of models with two moduli. As already argued in [19], in presence of one modulus only, non-vanishing tree-level gaugino masses would require  $f_v$  to depend on  $U$ . Due to the non-linear transformation of  $U$  under  $U(1)_X$ , anomalous transformations of the action are induced, which must be compensated by  $U(1)_X$ - $G_{\text{vis}}^2$  anomalies in the fermion spectrum, requiring  $q_{iv} \neq 0$  or some other modification, such as the introduction of  $U(1)_X$  charged fields, vector-like with respect to  $G_{\text{vis}}$ , which can also be seen as messenger fields of a high scale gauge mediation. This possibility — that would require to study the backreaction of the messengers on our vacuum — has been proposed in [19] to alleviate the hierarchy between the scalar and gaugino masses. We simply notice that in presence of two moduli, a more economical choice is to assume  $f_v$  to depend on the neutral modulus  $Z$  only, in which case one can safely take  $q_{iv} = 0$ . We will see in a specific example in the next section that this choice, together with  $\gamma_Z \neq 0$ , gives rise to a fully satisfactory scenario for gaugino and scalar mass terms.

The gaugino masses in the models with  $w_0 = 0$  sensitively depend on how we choose the exponential term  $\gamma$  in the up-lifting term. As we have seen, a natural up-lifting term requires  $\gamma_U \neq 0$  and hence  $\gamma_Z = 0$  if we allow the non-perturbatively generated coupling to depend on one modulus only. Furthermore, if we want to avoid introducing additional  $U(1)_X$  charged fields, then  $f_v$  should depend on  $Z$  only. In this case eq. (3.4) gives, for linearly moduli dependent  $f_v$ :

$$m_g \sim m_{3/2} \epsilon^2 X \left( \gamma_U + \frac{q_X}{X} \right) \sim m_{3/2} \epsilon, \tag{4.6}$$

where in the last relation the scaling  $\gamma_U \sim a$  has been used. The gaugino masses are significantly lighter than the gravitino now, so that anomaly mediated contributions cannot be neglected. Gaugino masses can be increased by allowing  $\gamma_Z$  to be non-vanishing, in which case they scale as in eq. (4.5). If one allows the non-perturbatively generated up-lifting term to depend on both moduli, then  $\gamma_U$  and  $\gamma_Z$  can respectively solve the naturalness problem of the up-lifting coupling and alleviate the modest hierarchy between gaugino and scalar masses.

## 5. Explicit models

The exponential sensitivity of the superpotential (2.15) on the moduli, the not so small value of the expansion parameter  $\epsilon \sim 1/30$  and the several other approximations made before do not generally allow for a reliable, quantitative analytical study of the theory. Indeed, the main aim of sections 2 and 3 was to qualitatively characterize the models and to show the existence of metastable Minkowski vacua with low-energy SUSY breaking in a large area in parameter and moduli space, rather than quantitatively study them. Aim of this section is to study at a more quantitative level three specific models, two with  $w_0 \neq 0$  and one with  $w_0 = 0$ . Most of the analysis here is performed numerically, because the exponential nature of the superpotential and the smallness of the  $D_X$  term at the minimum require a detailed knowledge of the location of the vacuum, in particular in the moduli directions. In order to appreciate this point, we will report in tables 1, 2 and 3 various quantities of interest computed starting both by the exact numerical vacuum and the approximate analytical one. The latter is found along the lines of subsection 2.1. We start from the SUSY configuration for  $\phi$ ,  $U$  and  $Z$  given by eqs. (2.14) and (2.22) for  $w_0 \neq 0$ , and eqs. (2.14), (3.1) and (3.9) for  $w_0 = 0$ . We then expand  $V_0$ , taking  $V_{up-lift}$  as a perturbation, around the SUSY vacuum, keeping only the linear terms in  $\Delta U$ ,  $\Delta Z$  and  $\Delta\phi$ . In this way we get what we denoted by  $U_0$ ,  $Z_0$  and  $\phi_0$ . We finally compute the VEV of  $\chi$  as explained below eq. (2.32).

Notice that even the numerical search of exact minima in presence of the  $D_X$ -term is not straightforward. The  $D_X$ -term is naturally of order one when slightly off-shell, and thus much bigger than the typical values of the  $F$  terms, namely one has  $V_D \gg V_F$  and all the energy of the system is dominated by  $V_D$ , hiding completely the stabilization of the moduli encoded in  $V_F$ . On the contrary, in the heavy gauge field approximation,  $V_D \ll V_F$  at the minimum, being  $O(F^4)$ , see eq. (2.25), which means that a severe fine-tuning takes place in  $V_D$  at the minimum. We have been able to circumvent this problem using a linear combination of the equations of motion for  $\phi$  and  $\chi$  to solve for  $D_X$  in terms of  $F$ -terms and their derivatives, and replace the result in these. We also found useful, instead of solving the equation of motion for  $\phi$ , to impose that the expression found for  $D_X$  to be equal to its expression (2.13) in terms of the fields. The numerical vacua so obtained turns out to be stable and the resulting  $D_X$  and  $F$ -terms always satisfies the consistency condition (2.11).

### 5.1 IIB model

The first model we consider is based on an orientifold compactification of type IIB string theory on a Calabi-Yau 3-fold obtained as an hyper-surface in  $\mathbf{CP}^4$ , namely  $\mathbf{CP}^4_{[1,1,1,6,9]}$ . This Calabi-Yau has  $h^{1,1} = 2$  and  $h^{2,1} = 272$  Kähler and complex structure moduli, respectively. See [35] for details. In the spirit of [2], we assume here that a combination of NSNS and RR fluxes stabilize the dilaton and complex structure moduli supersymmetrically. Once integrated out, these fields just give rise to a constant superpotential term  $w_0$ . We do not specify the detailed string construction which might give rise to the superpotential (2.3). We generally assume that  $D_7$  branes (and  $O_7$  planes) must be introduced to generate the non-perturbative superpotential terms in (2.3), as well as the non-linear trans-

formation under  $U(1)_X$  of the modulus  $U$  [36, 37]. We will neglect in the following possible open string moduli and consider the dynamics of the two Kähler moduli only, identifying them with the two moduli  $U$  and  $Z$ . We have now to specify the explicit form of the various terms entering in the Kähler potential (2.2). The purely moduli-dependent function  $K_M$  is known. In the usual approximation of neglecting flux effects, it takes the form [35]

$$K_M = -2 \log \text{Vol}, \quad \text{Vol} = \frac{1}{9\sqrt{2}} \left( \left( \frac{U + \bar{U}}{2} \right)^{3/2} - \left( \frac{Z + \bar{Z}}{2} \right)^{3/2} \right), \quad (5.1)$$

where Vol is the volume of the Calabi-Yau manifold. We do not specify the modular functions  $\alpha_{1,2}$  for the mesons  $M_{1,2}$  since they do not play any role in the limit where the mesons are supersymmetrically integrated out. The modular functions for  $\phi$  and  $\chi$ ,  $\alpha_\phi$  and  $\alpha_\chi$  in eq. (2.2), are instead relevant but are generally difficult to derive and depend on the underlying string construction. We assume here the following ansatz:

$$\alpha_\phi = \alpha_\chi = \frac{(Z + \bar{Z})}{\text{Vol}}, \quad (5.2)$$

which is simple enough, but not totally trivial. It should be stressed that there is nothing peculiar in the (arbitrary) choice we made in eq. (5.2). Any other choice will be fine as well, provided that  $\alpha_\chi$  is not too small. Indeed, according to eq. (2.32),  $\chi_0 \sim \alpha_\chi^{-1/2}$  and a sufficiently small  $\alpha_\chi$  can lead to a breakdown of our analysis based on an expansion in  $\chi$ . We have now to specify the various parameters entering in the hidden superpotential (2.3) and the gauge kinetic functions (2.4). Their choice is somehow arbitrary, but we require that  $U$  and  $Z$  and the volume of the Calabi-Yau to be sufficiently large to trust the classical SUGRA analysis. We take, for  $\eta_1 = -\eta_2 = -1$ ,<sup>11</sup>

$$\begin{aligned} N_1=40, & \quad N_{1f}=4, & \quad q_1=1, \quad c_1=1, & \quad p_1=0, & \quad n_1=\frac{1}{4\pi}, & \quad m_1=0, & \quad \eta_{1,U}=\eta_{1,Z}=0, \\ N_2=25, & \quad N_{2f}=1, & \quad q_2=0, \quad c_2=1, & \quad p_2=0, & \quad n_2=0, & \quad m_2=\frac{1}{4\pi}, & \quad \eta_{2,U}=\eta_{2,Z}=0, \\ w_0=9 \times 10^{-14}, & \quad Y=4.2 \times 10^{-5}, & \quad q_\chi=6, \quad \gamma_Z=\frac{1}{60}, & \quad \gamma_U=0, & \quad n_X=\frac{1}{4\pi}. \end{aligned} \quad (5.3)$$

The exponential moduli dependence in the up-lifting term  $V_{up-lift}$  is supposed to arise from some non-perturbative effect, such as stringy instantons [38]. The supersymmetric vacuum when the  $\chi$ -sector is turned off is

$$U_{\text{SUSY}} = 229, \quad Z_{\text{SUSY}} = 145, \quad \phi_{\text{SUSY}} = 6.3 \times 10^{-2}. \quad (5.4)$$

We report in table (1) (left panel) the location of the non-SUSY vacuum, as exactly found numerically and analytically by linearly expanding around the SUSY solution (5.4), as well as the  $F$ -terms,  $D_X$  and the potential  $V$  at the minimum. It can be seen that  $U$ ,  $Z$  and  $\phi$  are well reproduced analytically, whereas  $\chi$  is not, since its VEV is exponentially

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<sup>11</sup>What actually matters are the values of the phenomenological parameters (2.16), which do not uniquely fix the microscopical ones, as is evident from eq. (2.16). The choice (5.3) is purely illustrative. The same comment also applies to the next two examples below.

|                          | Numerical              | Analytical             |
|--------------------------|------------------------|------------------------|
| $\langle U \rangle$      | 232                    | 230                    |
| $\langle Z \rangle$      | 148                    | 146                    |
| $\langle \phi \rangle$   | $6.2 \times 10^{-2}$   | $6.3 \times 10^{-2}$   |
| $\langle \chi \rangle$   | $2.2 \times 10^{-4}$   | $1.3 \times 10^{-4}$   |
| $\langle F_U \rangle$    | $-4.0 \times 10^{-16}$ | $-2.3 \times 10^{-16}$ |
| $\langle F_Z \rangle$    | $4.2 \times 10^{-16}$  | $2.3 \times 10^{-16}$  |
| $\langle F_\phi \rangle$ | $8.5 \times 10^{-15}$  | $5.1 \times 10^{-15}$  |
| $\langle F_\chi \rangle$ | $2.1 \times 10^{-13}$  | $2.2 \times 10^{-13}$  |
| $\langle D_X \rangle$    | $1.5 \times 10^{-26}$  | $1.6 \times 10^{-27}$  |
| $\langle V \rangle$      | $9.2 \times 10^{-33}$  | $2.2 \times 10^{-32}$  |

|                            | Numerical            | Analytical           |
|----------------------------|----------------------|----------------------|
| $m_{3/2}$                  | 1.6                  | 1.6                  |
| $m_\phi$                   | $7.3 \times 10^{13}$ | $7.4 \times 10^{13}$ |
| $m_{\text{Re}(\chi)}$      | 242                  | 254                  |
| $m_{\text{Im}(\chi)}$      | 241                  | 253                  |
| $m_{\text{Im}(\tilde{U})}$ | 140                  | 176                  |
| $m_{\text{Re}(\tilde{Z})}$ | 97                   | 156                  |
| $m_{\text{Im}(\tilde{Z})}$ | 95                   | 105                  |
| $m_{\text{Re}(\tilde{U})}$ | 61                   | 93                   |

**Table 1:** VEVs, masses and scales for the IIB model with  $w_0 \neq 0$  and parameters given by eq. (5.3). Expectation values are expressed in (reduced) Planck units and masses in TeV units.  $\tilde{U} \sim U + Z$  and  $\tilde{Z} \sim U - Z$  stand for the (approximate) eigenvector mass states. The definitions are slightly different in the numerical and analytical cases due to the diagonalization of the kinetic terms.

sensitive to the values of  $U$  and  $Z$  by means of the non-perturbative terms in  $W_{\text{eff}}$ . For the same reason  $F_U$ ,  $F_Z$  and  $F_\phi$  are only roughly reproduced.  $F_\chi$ , instead, is better estimated since it essentially depends on  $Z$  only through the mild exponential appearing in the first term in eq. (2.6). The  $D_X$  term is also well reproduced because at leading order it is governed by  $F_\chi$  only, see eq. (2.25). In table (1) (right panel) the gravitino and all the scalar masses are reported, in TeV units. For simplicity of presentation, we have not written the precise linear combination of mass eigenvectors, but just the main components in field space. As anticipated, there is a hierarchy of scales. Fixing the overall scale such that  $m_{3/2} \simeq O(1)\text{TeV}$ , the field  $\phi$  (and the  $U(1)_X$  gauge boson  $A_X$ ) is ultra-heavy, whereas the moduli and  $\chi$  have masses  $O(100)$  TeV. Like for the F-terms, the masses which do not directly depend on the strong dynamics, namely  $m_{3/2}$ , whose mass is governed by  $w_0$ , and  $m_\phi$ , whose mass is well approximated by eq. (2.35), are well predicted analytically. In agreement with our general observations,  $1/(a_1 U) \simeq 1/(b_2 Z) \simeq 1/37 \equiv \epsilon$ . It is easy to check that the values of  $F_\phi$ ,  $F_U$  and  $F_Z$  reported in table (1) agree with the scaling behaviors predicted by eq. (2.31). As observed in subsection 2.1, the stability of the system requires a low value for  $\gamma_Z$  and hence the exponential dependence on  $Z$  of the coupling  $Y(U, Z, \phi)$  does not help much in getting a not so small  $Y$ . This constraints us to choose a rather large value of the  $U(1)_X$  charge of  $\chi$ ,  $q_\chi = 6$ , although such choice might not naturally appear in simple  $D$ -brane constructions. We can also compute the universal gaugino masses at the high scale, assuming  $f_v = f_2 = m_2 Z$ . We get

$$m_g \simeq 380 \text{ GeV}, \quad (5.5)$$

which is roughly one quarter the gravitino mass. As explained before, we assume  $U(1)_X$  neutral visible matter fields, so that the non-holomorphic soft scalar masses  $m \sim m_{3/2} \sim O(1)$  TeV, instead of  $m \simeq g_X \sqrt{D_X q_{iv}} \simeq 70 \text{ TeV} \sqrt{q_{iv}}$ , valid for  $U(1)_X$  charged fields.

We have finally considered the reliability of the SUGRA approximation by considering

the  $\alpha'$  correction appearing in  $K_M$ . For type IIB orientifolds, this is known to be [24]

$$\text{Vol} \rightarrow \text{Vol} + \frac{\xi}{2g_s^{3/2}}, \quad \xi = -\frac{\chi(M)\zeta(3)}{2(2\pi)^3}, \quad (5.6)$$

with  $\chi(M)$  the Euler characteristic of the Calabi-Yau and  $\zeta(3) = \sum_{n=1}^{\infty} 1/n^3 \simeq 1.2$ . In writing eq. (5.6), we have frozen the dilaton field  $S$ , which is taken non-dynamical and stabilized by fluxes to some value  $g_s = \text{Re } S$ . We have studied how the correction (5.6) roughly affects the model given by the input parameters (5.3), which are kept all fixed with the exception of  $Y$ , which is tuned to get an approximately Minkowski vacuum. A sizable correction is expected when  $\xi/2g_s^{3/2} \sim \text{Vol}$ . This is indeed the case, although we have numerically checked that the  $\alpha'$  correction is non-negligible already when it is  $O(\text{Vol}/10)$ . In our example  $\chi(\mathbf{CP}_{[1,1,1,6,9]}^4) = -540$  and for the vacuum shown in table (1), roughly speaking,  $g_s \gtrsim 1/10$  in order to trust the SUGRA analysis and neglect the correction (5.6).

## 5.2 Heterotic model

The second model we consider is inspired by a generic compactification of perturbative heterotic string theory on a Calabi-Yau 3-fold. In such a case, we identify  $U$  and  $Z$  with the dilaton and the universal Kähler modulus, respectively.<sup>12</sup> Contrary to the IIB case, unfortunately we do not still have a scenario to stabilize in a controlled way the complex structure moduli in the heterotic string. In addition to that, the presence of a small non vanishing constant superpotential term  $w_0$  is known to be not easily produced. Indeed, a flux for  $H$  induces a constant term in the superpotential [39], but being the  $H$  flux quantized, such a constant term is typically of order one in Planck units [40]. In the spirit of our bottom-up SUGRA approach, we do not look for a microscopic explanation for  $w_0$ . It might be the left-over term of the  $F$  and  $D$  terms vanishing conditions for all the extra fields which are supposed to occur in any realistic model, or else the left-over of a flux superpotential of an heterotic string compactified on generalized half-flat manifolds and non-standard embedding, which has been argued to admit quantized fluxes resulting in a small  $w_0$  [41]. The classical Kähler potential for the moduli is known to be [42]

$$K_M = -\log(U + \bar{U}) - 3\log(Z + \bar{Z}). \quad (5.7)$$

The modular functions  $\alpha_\phi$  and  $\alpha_\chi$  are now functions of  $Z$  only and typically expected to be of the form  $(Z + \bar{Z})^{-n}$ , with  $-1 \leq n \leq 0$ . We take here  $n = -1/3$ , so that

$$\alpha_\phi = \alpha_\chi = \frac{1}{(Z + \bar{Z})^{1/3}}. \quad (5.8)$$

The parameters entering in the gauge kinetic functions (2.4) and the superpotential (2.3) are quite more constrained with respect to the IIB case. At tree-level  $f_i = f_X = U$ . A possible  $Z$ -dependence can (and generally does) occur only at loop level by means of moduli-dependent threshold corrections. The exponential moduli dependence of the couplings  $Y$

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<sup>12</sup>In a more conventional notation the dilaton and universal Kähler modulus are denoted by  $S$  and  $T$ , respectively.



|                          | Numerical              | Analytical             |
|--------------------------|------------------------|------------------------|
| $\langle U \rangle$      | 1.78                   | 1.78                   |
| $\langle Z \rangle$      | 1.20                   | 1.20                   |
| $\langle \phi \rangle$   | 0.14                   | 0.14                   |
| $\langle \chi \rangle$   | $-3.2 \times 10^{-4}$  | $-2.0 \times 10^{-4}$  |
| $\langle F_U \rangle$    | $-1.9 \times 10^{-16}$ | $-1.5 \times 10^{-16}$ |
| $\langle F_Z \rangle$    | $-1.4 \times 10^{-15}$ | $-1.0 \times 10^{-15}$ |
| $\langle F_\phi \rangle$ | $-2.1 \times 10^{-17}$ | $-7.2 \times 10^{-18}$ |
| $\langle F_\chi \rangle$ | $4.0 \times 10^{-15}$  | $4.2 \times 10^{-15}$  |
| $\langle D_X \rangle$    | $5.4 \times 10^{-28}$  | $5.9 \times 10^{-28}$  |
| $\langle V \rangle$      | $1.3 \times 10^{-32}$  | $1.7 \times 10^{-32}$  |

|                    | Numerical            | Analytical           |
|--------------------|----------------------|----------------------|
| $m_{3/2}$          | 1.0                  | 1.0                  |
| $m_\phi$           | $3.1 \times 10^{14}$ | $3.1 \times 10^{14}$ |
| $m_\chi$           | 191                  | 200                  |
| $m_{\text{Re}(U)}$ | 98                   | 115                  |
| $m_{\text{Im}(U)}$ | 87                   | 107                  |
| $m_{\text{Im}(Z)}$ | 56                   | 61                   |
| $m_{\text{Re}(Z)}$ | 42                   | 52                   |

**Table 2:** VEVs, masses and scales for the heterotic model with  $w_0 \neq 0$  and parameters given by eq. (5.9). Expectation values are expressed in (reduced) Planck units and masses in TeV units.

and  $c_i$  is assumed to arise from world-sheet instantons and hence they depend on  $Z$  only. Finally, the size of the hidden gauge groups is bounded. We will look for vacua with  $U, Z \gtrsim 1$ , which are on the edge of perturbativity, but lie in the phenomenologically most interesting region in moduli space in perturbative heterotic theory. In fact, one should require  $\text{Re} U \sim 2$  for a successful SUSY GUT model, but since the 10-dimensional string coupling is given by  $g_s = \sqrt{Z^3/U}$  [42], perturbativity of the 10d heterotic string also requires  $Z^3 < U$  and hence  $Z \gtrsim 1$ . We then take

$$\begin{aligned}
 N_1=5, \quad N_{1f}=4, \quad q_1=1, \quad c_1=\frac{1}{2}, \quad \eta_{1,Z}=2\pi+\frac{3}{4}, \quad \eta_{1,U}=0, \quad m_1=0, \quad n_1=1, \quad p_1=0 \\
 N_2=4, \quad N_{2f}=2, \quad q_2=2, \quad c_2=1, \quad \eta_{2,Z}=0, \quad \eta_{2,U}=0, \quad m_2=0, \quad n_2=1, \quad p_2=0 \\
 w_0=3 \times 10^{-15}, \quad q_\chi=10, \quad Y=3.1 \times 10^{-3}, \quad \gamma_Z=2\pi, \quad \gamma_U=0, \quad n_X=1.
 \end{aligned} \tag{5.9}$$

The supersymmetric vacuum is at

$$U_{\text{SUSY}} = 1.76, \quad Z_{\text{SUSY}} = 1.19, \quad \phi_{\text{SUSY}} = 0.14. \tag{5.10}$$

In the heterotic case, being the tree-level gauge kinetic functions  $U$ -dependent only ( $m_{1,2} = 0$ ), one cannot have both  $\eta_{1,Z} = \eta_{2,Z} = 0$ , since this would lead to the vanishing of the effective parameters  $b_{1,2}$  defined in eq. (2.16), which is unacceptable. The particular choice of  $\eta_{1,Z}$  in eq. (5.9) is required to fix  $Z$  in the small window  $1 \lesssim Z \lesssim U$ , but of course there are several ways to achieve it in terms of the microscopic parameters, given that what matters are the effective ones defined in eq. (2.16). The asymmetry between  $U$  and  $Z$  in this heterotic inspired model does not allow to straightforwardly use the general scalings discussed in section 2.1. We do not report the analytical more elaborated analysis which is now required. One can nevertheless check that the general scalings (2.28) and (2.31) still give the rough order of magnitude estimate for the shifts of the fields and the  $F$  terms by taking  $\epsilon \simeq 1/(a_1 U) \simeq 1/28$ . We report in table (2) (left and right panel) the location of the non-SUSY vacuum, the  $F$ -terms,  $D_X$ , the potential and the masses for the scalars and the gravitino of the model. As in the IIB model before,  $\chi$  is not analytically reproduced

to a good accuracy, being exponentially sensitive to the values of  $U$  and  $Z$ . Similarly for  $F_U$ ,  $F_Z$  and  $F_\phi$ . The hierarchy of scales appearing in the gravitino and scalar masses are the same as in the Type IIB model. The large  $U(1)_X$  charge of  $\chi$ ,  $q_\chi = 10$ , allows for a natural explanation of the smallness of the up-lifting term without using tiny values for  $Y$ , as was done in (5.3) in the previous example. The tree-level universality of the heterotic holomorphic gauge kinetic functions (for level one Kac-Moody gauge groups, assumed here) fixes  $f_v = U$  and hence the universal high scale gaugino masses are calculable and read

$$m_g \simeq 240 \text{ GeV} . \tag{5.11}$$

Unfortunately it is not possible now to assume all visible matter fields to be  $U(1)_X$  neutral, since  $f_v = U$ . For the  $U(1)_X$  charged fields we get now  $m \simeq g_X \sqrt{D_X q_{iv}} \simeq 42 \text{ TeV} \sqrt{q_{iv}}$ .

The vacuum reported in table (2) is barely perturbative since the associated 10d string coupling constant  $g_s \simeq 1$ . One can explicitly see that the situation is similar in the  $\alpha'$  expansion, due to the low values of the moduli, by considering again the universal  $\alpha'$  correction to the Kähler potential for  $Z$ , which now reads [23]

$$(Z + \bar{Z})^3 \rightarrow (Z + \bar{Z})^3 + 4\xi, \tag{5.12}$$

with  $\xi$  defined as in eq. (5.6). We find that the  $\alpha'$  correction (5.12) is generally negligible for  $|\xi| \lesssim O(1/10)$  and are deadly for  $|\xi| \gtrsim O(1)$ , with the impossibility of achieving a Minkowski vacuum ( $\xi < 0$ ) or the appearance of tachyons ( $\xi > 0$ ). In the range  $O(1/10) \lesssim |\xi| \lesssim O(1)$  the qualitative properties of the model are unaffected, but the numerical values reported in table (2) get corrections of order 100%. Considering that for  $|\chi(M)| \sim 10^2$ , a typical value for Calabi-Yau manifolds,  $|\xi| \sim O(1)$ , it is clear that the phenomenologically interesting region  $U, Z \gtrsim 1$  is barely calculable, as we anticipated.

### 5.3 IIB model with $w_0 = 0$

As we have seen, the requirement of a natural  $Y$  favors the choice  $\gamma_U \neq 0$  for models with  $w_0 = 0$ . In an heterotic context, where  $U$  is the dilaton, we would be forced to invoke exotic non-perturbative couplings. In addition to that, hierarchies would also appear in the mesonic Yukawa couplings  $c_i$ . The upper bound on the gauge groups leads to a lower bound on  $a$ ,  $a \gtrsim 15$  and  $a_1 - a_2 \sim 3$ . Using eq. (3.1), it is easy to see that the phenomenological requirement  $U \sim 2$  constraints  $A_1/A_2 \sim 10^3$ . In light of eq. (2.16), this hierarchy in  $A_1/A_2$  typically induces an even larger hierarchy in the microscopical Yukawa couplings  $c_i$ , unless one assumes a hierarchy between them due to, say, world-sheet instantons. For these reasons, as a specific example of model with  $w_0 = 0$ , we again opt here for a type IIB model on  $\mathbf{CP}^4_{[1,1,1,6,9]}$ , with  $U$  and  $Z$  the two Kähler moduli of the Calabi-Yau manifold. No bound on the gauge group arises and the situation seems more favorable. For simplicity we take now trivial Kähler potentials for  $\phi$  and  $\chi$ , namely

$$\alpha_\chi = \alpha_\phi = 1 . \tag{5.13}$$

|                          | Numerical              | Analytical             |
|--------------------------|------------------------|------------------------|
| $\langle U \rangle$      | 136.0                  | 136.0                  |
| $\langle Z \rangle$      | 63.3                   | 63.3                   |
| $\langle \phi \rangle$   | 0.101                  | 0.101                  |
| $\langle \chi \rangle$   | $-3.6 \times 10^{-6}$  | $-7.6 \times 10^{-7}$  |
| $\langle F_U \rangle$    | $-2.7 \times 10^{-17}$ | $-2.6 \times 10^{-17}$ |
| $\langle F_Z \rangle$    | $-7.2 \times 10^{-18}$ | $-8.1 \times 10^{-18}$ |
| $\langle F_\phi \rangle$ | $1.5 \times 10^{-16}$  | $1.6 \times 10^{-16}$  |
| $\langle F_\chi \rangle$ | $2.0 \times 10^{-13}$  | $2.0 \times 10^{-13}$  |
| $\langle D_X \rangle$    | $1.2 \times 10^{-26}$  | $1.2 \times 10^{-27}$  |
| $\langle V \rangle$      | $2.4 \times 10^{-32}$  | $2.4 \times 10^{-32}$  |

|                            | Numerical            | Analytical           |
|----------------------------|----------------------|----------------------|
| $m_{3/2}$                  | 3.31                 | 3.31                 |
| $m_\phi$                   | $1.1 \times 10^{14}$ | $1.1 \times 10^{14}$ |
| $m_{\text{Im}(\tilde{U})}$ | $6.3 \times 10^3$    | $6.3 \times 10^3$    |
| $m_{\text{Re}(\tilde{U})}$ | $6.3 \times 10^3$    | $6.3 \times 10^3$    |
| $m_{\text{Re}(\tilde{Z})}$ | 229                  | 229                  |
| $m_{\text{Im}(\tilde{Z})}$ | 212                  | 212                  |
| $m_{\text{Im}(\chi)}$      | 160                  | 160                  |
| $m_{\text{Re}(\chi)}$      | 160                  | 160                  |

**Table 3:** VEVs, masses and scales for the IIB model with  $w_0 = 0$  and parameters given by eq. (5.14). Expectation values are expressed in (reduced) Planck units and masses in TeV units.  $\tilde{U} \sim U + Z$  and  $\tilde{Z} \sim U - Z$  stand for the (approximate) eigenvector mass states.

The input parameters are as follows:

$$\begin{aligned}
 N_1=30, \quad N_{1f}=2, \quad q_1=2, \quad c_1=1, \quad p_1=\frac{18}{50}, \quad n_1=\frac{1}{4\pi}, \quad m_1=0, \quad \eta_{1,U}=\eta_{1,Z}=0, \\
 N_2=29, \quad N_{2f}=2, \quad q_2=2, \quad c_2=2, \quad p_2=0, \quad n_2=\frac{1}{4\pi}, \quad m_2=0, \quad \eta_{2,U}=\eta_{2,Z}=0, \\
 N_3=11, \quad N_{3f}=1, \quad q_3=0, \quad c_2=1, \quad p_3=0, \quad n_3=0, \quad m_3=\frac{1}{4\pi}, \quad \eta_{3,U}=\eta_{3,Z}=0, \\
 Y=\frac{867}{5000}, \quad q_\chi=2, \quad n_X=\frac{1}{4\pi}, \quad \gamma_Z=0, \quad \gamma_U=\frac{1}{6}. \tag{5.14}
 \end{aligned}$$

The supersymmetric vacuum is at

$$U_{\text{SUSY}} = 136, \quad Z_{\text{SUSY}} = 63, \quad \phi_{\text{SUSY}} = 0.10, \tag{5.15}$$

and the exact non-SUSY vacuum, and its properties, is reported in table(3). Notice that the model is quite constrained. Given  $N_{1,2}$ ,  $N_{1f,2f}$  and  $q_{1,2}$ , for mesonic Yukawa couplings  $c_i \sim O(1)$ , the gauge kinetic functions are essentially fixed by the requirement of low-energy SUSY. A constant threshold correction, appearing in (5.14), can be avoided by allowing a mild tuning between  $c_1$  and  $c_2$ . The value of the coupling  $Y$  is fixed by the flat condition. As expected from the general arguments of section 4, the gauginos are now light and indeed we get, for  $f_v = f_3 = Z/(4\pi)$ ,

$$m_g \simeq 270 \text{ GeV}, \tag{5.16}$$

which is less than one order of magnitude smaller than  $m_{3/2}$ . We can take  $U(1)_X$  neutral visible matter fields, so that the non-holomorphic soft scalar masses  $m \simeq m_{3/2}$ , instead of  $m \simeq g_X \sqrt{D_X q_{iv}} \simeq 80 \text{ TeV} \sqrt{q_{iv}}$ , valid for  $U(1)_X$  charged fields.

The analytical and numerical values in this case agree with very good accuracy thanks to the smallness of  $\chi$ . We have  $1/(b_3 Z) \simeq 1/36 \sim 1/(a_{1,2} U) \simeq 1/30$  and the expected scalings for the shifts of the fields and the  $F$  terms are satisfied. The value of  $\chi$  as would be roughly predicted by eq. (3.6) is about one order of magnitude bigger than its actual value,

due to accidental numerical factors such that  $q_\chi \chi F_\chi \sim \phi F_\phi / 10$ . Notice how this hybrid model, where the KKLT-like and racetrack stabilizations work together, has all the appealing features we were looking for. The stabilization via the racetrack mechanism of  $U$  allows to get a natural Yukawa coupling  $Y$ , and the stabilization of  $Z$  after the uplifting generates acceptable gaugino masses and avoid the problem of having too heavy scalar soft masses.

We have numerically checked the reliability of the classical SUGRA analysis by looking at the  $\alpha'$  correction (5.6). As expected, the correction becomes sizable for  $|\xi|/g_s^{3/2} \gtrsim \text{Vol}$ , i.e.,  $g_s \sim 1/25$  and is essentially negligible for  $|\xi|/g_s^{3/2} \lesssim \text{Vol}/10$ . This results on a mild lower bound on the string coupling:  $g_s \gtrsim 1/6$ . Contrary to the case with  $w_0 \neq 0$  where the possibility of getting a Minkowski vacuum is lost for  $g_s$  smaller than the bound, no dramatic consequences appear now, in the sense that the corrections are only quantitative, but the non-SUSY Minkowski vacuum is still there, even for  $g_s \sim 1/30$ .<sup>13</sup>

## 6. Conclusions

We have shown in this paper how to get realistic, string inspired, SUGRA Minkowski/dS vacua with two moduli, where the SUSY breaking mechanism is induced by a FI term appearing in the  $D$  term of a would-be anomalous  $U(1)_X$  gauge symmetry. Like in the original Fayet model, the FI coupling and the presence of a superpotential term  $\chi \phi^{q_\chi}$  for two charged fields  $\phi$  and  $\chi$  force the system to break SUSY. Being the FI term moduli-dependent, the stabilization of the moduli is a crucial (and anyway important) ingredient in the framework. The mechanism can be made stable and relatively natural by invoking a ratio of  $U(1)_X$  charges between  $\phi$  and  $\chi$  by almost one order of magnitude. When the moduli are stabilized without a constant superpotential term  $w_0$ , the mechanism is more robust and no bound on the  $U(1)_X$  charges arises. The moduli masses are proportional to the scale of SUSY breaking and hence a gravity mediation of SUSY breaking, with a gravitino mass of  $O(\text{TeV})$ , is preferred for cosmological reasons.

Studying the dynamics of two moduli, instead of one only, is not just an academic complication, because in this set-up one modulus necessarily transforms under the  $U(1)_X$  gauge symmetry, whereas the second can be taken neutral. The two-moduli system is then the simplest scenario that can describe more realistic situations. Moreover, we have shown how the presence of the second neutral modulus can considerably help in getting sufficiently heavy gaugino masses without getting at the same time very heavy scalar masses, a property which is not so common in string compactifications.

All our analysis has been based on the search of non-SUSY solutions starting from SUSY ones. It should be stressed that this does not imply that our vacua are small deformations of SUSY ones, since the small parameter  $\epsilon$  is fixed by the Planck/weak scale hierarchy and cannot be continuously taken to be zero. On the other hand, we cannot exclude the existence of other interesting non-SUSY vacua which are not detectable with our approach. We think that the above SUSY breaking mechanism, together with the

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<sup>13</sup>In this and in the previous IIB example with  $w_0 \neq 0$ , the assumed decoupling between Kähler and dilaton moduli stabilization will give rise to a further constraint on  $g_s$ , since the  $\alpha'$  correction (5.6) introduce a mixing between the two sectors.

stabilization of moduli by non-perturbative gauge dynamics, is interesting, promising and can be made quite natural, particularly when  $w_0 = 0$ . The next crucial step, as always when considering bottom-up SUGRA phenomenological models, would be to embed this mechanism in a full string theory set-up.

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## References

- [1] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev. D* **66** (2002) 106006 [[hep-th/0105097](#)].
- [2] S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, *De Sitter vacua in string theory*, *Phys. Rev. D* **68** (2003) 046005 [[hep-th/0301240](#)].
- [3] K. Choi, A. Falkowski, H.P. Nilles, M. Olechowski and S. Pokorski, *Stability of flux compactifications and the pattern of supersymmetry breaking*, *JHEP* **11** (2004) 076 [[hep-th/0411066](#)];  
K. Choi, A. Falkowski, H.P. Nilles and M. Olechowski, *Soft supersymmetry breaking in KKLT flux compactification*, *Nucl. Phys. B* **718** (2005) 113 [[hep-th/0503216](#)].
- [4] M. Endo, M. Yamaguchi and K. Yoshioka, *A bottom-up approach to moduli dynamics in heavy gravitino scenario: superpotential, soft terms and sparticle mass spectrum*, *Phys. Rev. D* **72** (2005) 015004 [[hep-ph/0504036](#)].
- [5] G. Villadoro and F. Zwirner, *De Sitter vacua via consistent D-terms*, *Phys. Rev. Lett.* **95** (2005) 231602 [[hep-th/0508167](#)].
- [6] A. Achucarro, B. de Carlos, J.A. Casas and L. Doplicher, *De Sitter vacua from uplifting D-terms in effective supergravities from realistic strings*, *JHEP* **06** (2006) 014 [[hep-th/0601190](#)].
- [7] M. Gomez-Reino and C.A. Scrucca, *Locally stable non-supersymmetric Minkowski vacua in supergravity*, *JHEP* **05** (2006) 015 [[hep-th/0602246](#)].
- [8] S.L. Parameswaran and A. Westphal, *De Sitter string vacua from perturbative Kähler corrections and consistent D-terms*, *JHEP* **10** (2006) 079 [[hep-th/0602253](#)].
- [9] O. Lebedev, H.P. Nilles and M. Ratz, *De Sitter vacua from matter superpotentials*, *Phys. Lett. B* **636** (2006) 126 [[hep-th/0603047](#)].
- [10] K. Choi and K.S. Jeong, *Supersymmetry breaking and moduli stabilization with anomalous U(1) gauge symmetry*, *JHEP* **08** (2006) 007 [[hep-th/0605108](#)].
- [11] F. Brummer, A. Hebecker and M. Trapletti, *SUSY breaking mediation by throat fields*, *Nucl. Phys. B* **755** (2006) 186 [[hep-th/0605232](#)].
- [12] E. Dudas and Y. Mambrini, *Moduli stabilization with positive vacuum energy*, *JHEP* **10** (2006) 044 [[hep-th/0607077](#)].

- [13] E. Dudas, C. Papineau and S. Pokorski, *Moduli stabilization and uplifting with dynamically generated F-terms*, *JHEP* **02** (2007) 028 [[hep-th/0610297](#)].
- [14] H. Abe, T. Higaki, T. Kobayashi and Y. Omura, *Moduli stabilization, F-term uplifting and soft supersymmetry breaking terms*, *Phys. Rev. D* **75** (2007) 025019 [[hep-th/0611024](#)].
- [15] R. Kallosh and A. Linde, *O'KKLT*, *JHEP* **02** (2007) 002 [[hep-th/0611183](#)].
- [16] O. Lebedev, V. Lowen, Y. Mambrini, H.P. Nilles and M. Ratz, *Metastable vacua in flux compactifications and their phenomenology*, *JHEP* **02** (2007) 063 [[hep-ph/0612035](#)].
- [17] D. Cremades, M.P. Garcia del Moral, F. Quevedo and K. Suruliz, *Moduli stabilisation and de Sitter string vacua from magnetised D7 branes*, *JHEP* **05** (2007) 100 [[hep-th/0701154](#)].
- [18] Z. Lalak, O.J. Eyton-Williams and R. Matyszkiewicz, *F-term uplifting via consistent D-terms*, *JHEP* **05** (2007) 085 [[hep-th/0702026](#)].
- [19] E. Dudas, Y. Mambrini, S. Pokorski and A. Romagnoni, *Moduli stabilization with Fayet-Iliopoulos uplift*, *JHEP* **04** (2008) 015 [[arXiv:0711.4934](#)].
- [20] A. Achucarro and K. Sousa, *F-term uplifting and moduli stabilization consistent with Kähler invariance*, [arXiv:0712.3460](#).
- [21] P. Fayet and J. Iliopoulos, *Spontaneously broken supergauge symmetries and Goldstone spinors*, *Phys. Lett.* **B 51** (1974) 461;  
P. Fayet, *Spontaneously broken supersymmetric theories of weak, electromagnetic and strong interactions*, *Phys. Lett.* **B 69** (1977) 489.
- [22] N.V. Krasnikov, *On supersymmetry breaking in superstring theories*, *Phys. Lett.* **B 193** (1987) 37;  
For earlier phenomenological string inspired applications of this mechanism see e.g.  
J.A. Casas, Z. Lalak, C. Muñoz and G.G. Ross, *Hierarchical supersymmetry breaking and dynamical determination of compactification parameters by nonperturbative effects*, *Nucl. Phys.* **B 347** (1990) 243;  
B. de Carlos, J.A. Casas and C. Muñoz, *Supersymmetry breaking and determination of the unification gauge coupling constant in string theories*, *Nucl. Phys.* **B 399** (1993) 623 [[hep-th/9204012](#)].
- [23] P. Candelas, X.C. De La Ossa, P.S. Green and L. Parkes, *A pair of Calabi-Yau manifolds as an exactly soluble superconformal theory*, *Nucl. Phys.* **B 359** (1991) 21.
- [24] K. Becker, M. Becker, M. Haack and J. Louis, *Supersymmetry breaking and  $\alpha'$ -corrections to flux induced potentials*, *JHEP* **06** (2002) 060 [[hep-th/0204254](#)].
- [25] M.B. Green and J.H. Schwarz, *Anomaly cancellation in supersymmetric D = 10 gauge theory and superstring theory*, *Phys. Lett.* **B 149** (1984) 117;  
M. Dine, N. Seiberg and E. Witten, *Fayet-Iliopoulos terms in string theory*, *Nucl. Phys.* **B 289** (1987) 589.
- [26] V. Balasubramanian, P. Berglund, J.P. Conlon and F. Quevedo, *Systematics of moduli stabilisation in Calabi-Yau flux compactifications*, *JHEP* **03** (2005) 007 [[hep-th/0502058](#)];  
J.P. Conlon, F. Quevedo and K. Suruliz, *Large-volume flux compactifications: moduli spectrum and D3/D7 soft supersymmetry breaking*, *JHEP* **08** (2005) 007 [[hep-th/0505076](#)].

- [27] I. Affleck, M. Dine and N. Seiberg, *Dynamical supersymmetry breaking in supersymmetric QCD*, *Nucl. Phys. B* **241** (1984) 493; *Dynamical supersymmetry breaking in four-dimensions and its phenomenological implications*, *Nucl. Phys. B* **256** (1985) 557;  
T.R. Taylor, G. Veneziano and S. Yankielowicz, *Supersymmetric QCD and its massless limit: an effective Lagrangian analysis*, *Nucl. Phys. B* **218** (1983) 493.
- [28] M. Gomez-Reino and C.A. Scrucca, *Metastable supergravity vacua with F and D supersymmetry breaking*, *JHEP* **08** (2007) 091 [[arXiv:0706.2785](#)].
- [29] P. Binetruy and E. Dudas, *Gaugino condensation and the anomalous U(1)*, *Phys. Lett. B* **389** (1996) 503 [[hep-th/9607172](#)].
- [30] A.E. Nelson and N. Seiberg, *R symmetry breaking versus supersymmetry breaking*, *Nucl. Phys. B* **416** (1994) 46 [[hep-ph/9309299](#)].
- [31] N. Arkani-Hamed, M. Dine and S.P. Martin, *Dynamical supersymmetry breaking in models with a Green-Schwarz mechanism*, *Phys. Lett. B* **431** (1998) 329 [[hep-ph/9803432](#)].
- [32] M. Serone and A. Westphal, *Moduli stabilization in meta-stable heterotic supergravity vacua*, *JHEP* **08** (2007) 080 [[arXiv:0707.0497](#)].
- [33] L. Randall and R. Sundrum, *Out of this world supersymmetry breaking*, *Nucl. Phys. B* **557** (1999) 79 [[hep-th/9810155](#)];  
G.F. Giudice, M.A. Luty, H. Murayama and R. Rattazzi, *Gaugino mass without singlets*, *JHEP* **12** (1998) 027 [[hep-ph/9810442](#)].
- [34] V.S. Kaplunovsky and J. Louis, *Model independent analysis of soft terms in effective supergravity and in string theory*, *Phys. Lett. B* **306** (1993) 269 [[hep-th/9303040](#)].
- [35] F. Denef, M.R. Douglas and B. Florea, *Building a better racetrack*, *JHEP* **06** (2004) 034 [[hep-th/0404257](#)].
- [36] M. Haack, D. Krefl, D. Lüst, A. Van Proeyen and M. Zagermann, *Gaugino condensates and D-terms from D7-branes*, *JHEP* **01** (2007) 078 [[hep-th/0609211](#)].
- [37] H. Jockers and J. Louis, *D-terms and F-terms from D7-brane fluxes*, *Nucl. Phys. B* **718** (2005) 203 [[hep-th/0502059](#)].
- [38] R. Blumenhagen, M. Cvetič and T. Weigand, *Spacetime instanton corrections in 4D string vacua — the seesaw mechanism for D-brane models*, *Nucl. Phys. B* **771** (2007) 113 [[hep-th/0609191](#)];  
L.E. Ibáñez and A.M. Uranga, *Neutrino Majorana masses from string theory instanton effects*, *JHEP* **03** (2007) 052 [[hep-th/0609213](#)].
- [39] M. Dine, R. Rohm, N. Seiberg and E. Witten, *Glino condensation in superstring models*, *Phys. Lett. B* **156** (1985) 55.
- [40] R. Rohm and E. Witten, *The antisymmetric tensor field in superstring theory*, *Ann. Phys. (NY)* **170** (1986) 454.
- [41] B. de Carlos, S. Gurrieri, A. Lukas and A. Micu, *Moduli stabilisation in heterotic string compactifications*, *JHEP* **03** (2006) 005 [[hep-th/0507173](#)].
- [42] E. Witten, *Dimensional reduction of superstring models*, *Phys. Lett. B* **155** (1985) 151.